1. For a proportional hazards regression model with one explanatory variable, write the log partial likelihood, differentiate and set to zero.

   a) What happened to β_0?

   b) Simplify, writing $\ell' = 0$ as a sum that equals zero.

2. Prove $S(t) = e^{-H(t)}$ (again)

3. For the proportional hazards model, let

   $h(t|\beta) = h_0(t)\, e^{\beta_0 + x_i^T \beta}$. Prove

   $S(t) = S_0(t)^{\exp\{\beta_0 + x_i^T \beta\}}$

1.

$$L = \prod_{i=1}^{n} \frac{e^{\beta_0 + \beta_1 x_{(i)}}}{\sum_{j \in R_i} e^{\beta_0 + \beta_1 x_j}} = \frac{e^{\beta_0} e^{\beta_1 x_{(i)}}}{\sum_{j \in R_i} e^{\beta_0} e^{\beta_1 x_j}}$$

$$= \prod_{i=1}^{n} \frac{e^{\beta_1 x_{(i)}}}{\sum_{j \in R_i} e^{\beta_1 x_j}} = \frac{e^{\beta_1 \sum_{i=1}^{n} x_{(i)}}}{\prod_{i=1}^{n} \sum_{j \in R_i} e^{\beta_1 x_j}}$$

$$\ell = \beta_1 \sum_{i=1}^{n} x_{(i)} - \sum_{i=1}^{n} \log \sum_{j \in R_i} e^{\beta_1 x_j}$$

$$\frac{\partial \ell}{\partial \beta_1} = \sum_{i=1}^{n} x_{(i)} - \sum_{i=1}^{n} \frac{\sum_{j \in R_i} x_j e^{\beta_1 x_j}}{\sum_{j \in R_i} e^{\beta_1 x_j}} \overset{set}{=} 0$$

$$= \sum_{i=1}^{n} \left(x_{(i)} - \sum_{j \in R_i} x_j \frac{e^{\beta_1 x_j}}{\sum_{k \in R_i} e^{\beta_1 x_k}} \right)$$

$$\sum_{x} x \, p(x)$$

2. Prove $S(t) = e^{-H(t)}$

$$H(t) = \int_0^t h(x)\,dx = \int_0^t \frac{f(x)}{S(x)}\,dx$$

$$= -\int_0^t \frac{-f(x)}{1-F(x)}\,dx$$

$u = 1 - F(x)$

$du = -f(x)\,dx$

x	$u = 1-F(x) = S(x)$
t	$S(t)$
0	$1-F(0) = 1$

$$= -\int_1^{S(t)} \frac{1}{u}\,du$$

$$= -(\log S(t) - \log(1)) = -\log(S(t)) = H(t)$$

$$\Longrightarrow S(t) = e^{-H(t)}$$

3. Set $h(t|\beta) = h_0(t)\, e^{\beta_0 + x_i^T \beta}$

Show $S(t) = S_0(t)^{\exp\{\beta_0 + x_i^T \beta\}}$

$$S_0(t) = e^{-H_0(t)}$$

$$S(t) = e^{-H(t)} = e^{-\int_0^t h(x)\,dx}$$

$$= e^{-\int_0^t h_0(y) e^{\beta_0 + x_i^T \beta} dy}$$

$$= e^{-e^{\beta_0 + x_i^T \beta} \int_0^t h_0(y)\,dy}$$

$$= \left(e^{-\int_0^t h_0(y)\,dy}\right)^{e^{\beta_0 + x_i^T \beta}}$$

$$= S_0(t)^{e^{\beta_0 + x_i^T \beta}}$$

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