## STA 312s19 Assignment Seven ${ }^{1}$

The paper and pencil part of this assignment is not to be handed in. It is practice for Quiz 7 on March 4th. The R part may be handed in as part of the quiz. Bring hard copy of your printout to the quiz. Do not write anything on your printout in advance except possibly your name and student number.

1. High School History classes from across Ontario are randomly assigned to either a discovery-oriented or a memory-oriented curriculum in Canadian history. At the end of the year, the students are given a standardized test and the median score of each class is recorded. Please consider a regression model with these variables:
$X_{1}$ Equals 1 if the class uses the discovery-oriented curriculum, and equals 0 the class it uses the memory-oriented curriculum.
$X_{2}$ Average parents' education for the classroom
$X_{3}$ Average parents' income for the classroom
$X_{4}$ Number of university History courses taken by the teacher
$X_{5}$ Teacher's final cumulative university grade point average
$Y$ Class median score on the standardized history test.
The full regression model has $E[Y \mid \mathbf{x}]=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\beta_{3} x_{3}+\beta_{4} x_{4}+\beta_{5} x_{5}$. For each question below, give the null hypothesis in terms of $\beta$ values. Also give $E[Y \mid \mathbf{x}]$ for the restricted (reduced) model you would use to answer each question. Don't re-number the variables.
(a) If you control for parents' education and income and for teacher's university background, does curriculum type affect test scores?
(b) Controlling for parents' education and income and for curriculum type, is teacher's university background (two variables) related to students' test performance?
(c) Controlling for teacher's university background and for curriculum type, are parents' education and income (considered simultaneously) related to students' test performance?
(d) Controlling for curriculum type, teacher's university background and parents' education, is parents' income related to students' test performance?

[^0]2. In a study comparing the effectiveness of different exercise programmes, volunteers were randomly assigned to one of three exercise programmes $(A, B, C)$ or put on a waiting list and told to work out on their own. Aerobic capacity is the body's ability to process oxygen. Aerobic capacity was measured before and after 6 months of participation in the program (or 6 months of being on the waiting list). The response variable was improvement in aerobic capacity. The explanatory variables were age (a covariate) and treatment group. Consider a regression model with an intercept, and no interaction between age and treatment group.
(a) Make a table showing how you would set up indicator dummy variables for treatment group. Make Waiting List the reference category
(b) Write the regression model. Please use $x$ for age, and make its regression coefficient $\beta_{1}$.
(c) In terms of $\beta$ values, what null hypothesis would you test to find out whether, allowing for age, the three exercise programmes differ in their effectiveness?
(d) Write the null hypothesis for the preceding question as $H_{0}: \mathbf{L} \boldsymbol{\beta}=\mathbf{0}$. Just give the $\mathbf{L}$ matrix.
(e) In terms of $\beta$ values, what null hypothesis would you test to find out whether Programme $B$ was better than the waiting list?
(f) In terms of $\beta$ values, what null hypothesis would you test to find out whether Programmes $A$ and $B$ differ in their effectiveness?
(g) Suppose you wanted to estimate the difference in average benefit between programmes $A$ and $C$ for a 27 year old participant. Give your answer in terms of $\widehat{\beta}$ values.
(h) Is it safe to assume that age is independent of the other explanatory variables? Answer Yes or No and briefly explain.
3. Pigs are routinely given large doses of antibiotics even when they show no signs of illness, to protect their health under unsanitary conditions. Pigs were randomly assigned to one of three antibiotic drugs. Dressed weight (weight of the pig after slaughter and removal of head, intestines and skin) was the dependent variable. Independent variables are Drug type, Mother's live adult weight and Father's live adult weight.
Data are in the file pigweight.data.txt. You can get a copy with
oink = read.table("http://www.utstat.toronto.edu/~brunner/data/legal/pigweight.data.txt").
(a) Write the regression equation for the full model, including $\epsilon_{i}$.
(b) Make a table with one row for every drug, with columns showing how the dummy variables were defined. Make another column giving $E(y \mid \mathbf{x})$ for each drug.
(c) Predict the dressed weight of a pig getting Drug 2, whose mother weighed 140 pounds, and whose father weighed 185 pounds. Your answer is a single number.
(d) This parallel planes regression model specifies that the differences in expected weight for the different drug treatments are the same for every possible combination of mother's weight and father's weight. Give a $95 \%$ confidence interval for the difference in expected weight between drug treatments 2 and 3 . The final answer is a pair of numbers, a lower confidence limit and an upper confidence limit. There is an easy way and a less easy way. The lecture slides illustrate both ways.
(e) In symbols, give the null hypotheses you would test to answer the following questions. Your answers are statements involving the $\beta$ values from your regression equation.
i. Controlling for mother's weight and father's weight, does type of drug have an effect on the expected weight of a pig?
ii. Controlling for mother's weight and father's weight, which drug helps the average pig gain more weight, Drug 1 or Drug 2?
iii. Controlling for mother's weight and father's weight, which drug helps the average pig gain more weight, Drug 1 or Drug 3?
iv. Controlling for mother's weight and father's weight, which drug helps the average pig gain more weight, Drug 2 or Drug 3?
(f) For each of the questions below, give the value of the $t$ or $F$ statistic (a number from your printout), and indicate whether or not you reject the null hypothesis. The numbers may or may not be part of the default output from summary.
i. Controlling for mother's weight and father's weight, does type of drug have an effect on the expected weight of a pig?
ii. Controlling for mother's weight and father's weight, which drug helps the average pig gain more weight, Drug 1 or Drug 2?
iii. Controlling for mother's weight and father's weight, which drug helps the average pig gain more weight, Drug 1 or Drug 3?
iv. Controlling for mother's weight and father's weight, which drug helps the average pig gain more weight, Drug 2 or Drug 3?
v. Allowing for which drug they were given, does expected weight of a pig increase faster as a function of the mother's weight, or does it increase faster as a function of the father's weight?
(g) We can assume that farmers want their pigs to weigh a lot. In plain, non-statistical language, can you offer some advice to a farmer based on these data? Remember, the farmer must be able to understand your answer or it is worthless.

Please bring your printout to the quiz. Your printout should show all $\mathbf{R}$ input and output, and only $\mathbf{R}$ input and output. Do not write anything on your printouts except your name and student number.


[^0]:    ${ }^{1}$ This assignment was prepared by Jerry Brunner, Department of Mathematical and Computational Sciences, University of Toronto. It is licensed under a Creative Commons Attribution - ShareAlike 3.0 Unported License. Use any part of it as you like and share the result freely. The $\mathrm{IA}_{\mathrm{E}} \mathrm{X}$ source code is available from the course website: http://www.utstat.toronto.edu/~brunner/oldclass/312s19

