Within-cases analysis of Normal Data

The grapefruit data

Grapefruit sales were measured for eight grocery stores at three price levels. The multivariate data format is more natural, but `lmer` wants each value of the response variable to be on a separate line.

Multivariate format

```
 Store   sales1    sales2    sales3
   1      62.1      61.3      60.8
   2      58.2      57.9      55.1
   3      51.6      49.2      46.2
   4      53.7      51.5      48.3
   5      61.4      58.7      56.6
   6      58.5      57.2      54.3
   7      46.8      43.2      41.5
   8      51.2      49.8      47.9
```

Univariate format

```
 Store   Price   Sales
   1       1      62.1
   1       2      61.3
   1       3      60.8
   2       1      58.2
   2       2      57.9
   2       3      55.1
   3       1      51.6
   3       2      49.2
   3       3      46.2
   4       1      53.7
   4       2      51.5
   4       3      48.3
   5       1      61.4
   5       2      58.7
   5       3      56.6
   6       1      58.5
   6       2      57.2
   6       3      54.3
   7       1      46.8
   7       2      43.2
   7       3      41.5
   8       1      51.2
   8       2      49.8
   8       3      47.9
```

These data are from a homework problem in *Applied linear statistical models* (2005, 5th edition) by Kutner et al.

```
> ###################### Grapefruit ######################
> # http://www.utstat.utoronto.ca/~brunner/data/illegal/GrapefruitData.html
>
> rm(list=ls()); options(scipen=999) # To avoid scientific notation
> # Install packages if necessary. Only need to do this once.
> # install.packages("lme4")
> # install.packages("car")
> # Load packages -- do this every time
> library(lme4) # For lmer function
Loading required package: Matrix
> library(car)  # For F-tests, likelihood ratio and Wald chi-squared tests
> # Avoid the lmerTest package. It seems to have actual errors.
>
> fruit = read.table("grapefruit.data.txt",header=T) # Local copy
> head(fruit); attach(fruit)
  Store Price Sales
1     1     1  62.1
2     1     2  61.3
3     1     3  60.8
4     2     1  58.2
5     2     2  57.9
6     2     3  55.1
> table(Price,Store)
     Store
Price 1 2 3 4 5 6 7 8
    1 1 1 1 1 1 1 1 1
    2 1 1 1 1 1 1 1 1
    3 1 1 1 1 1 1 1 1
> Price = factor(Price) # Otherwise it's numeric
> aggregate(fruit,by=list(Price),FUN=mean)
  Group.1 Store Price   Sales
1       1   4.5     1 55.4375
2       2   4.5     2 53.6000
3       3   4.5     3 51.3375
> gfmixed = lmer(Sales ~ Price + (1 | Store) )
> summary(gfmixed)
Linear mixed model fit by REML ['lmerMod']
Formula: Sales ~ Price + (1 | Store)

REML criterion at convergence: 93.2

Scaled residuals:
     Min       1Q   Median       3Q      Max
-1.48527 -0.41034 -0.07546  0.53703  1.90090

Random effects:
 Groups   Name        Variance Std.Dev.
 Store    (Intercept) 35.2571  5.9378
 Residual              0.6837  0.8269
Number of obs: 24, groups:  Store, 8

Fixed effects:
            Estimate Std. Error t value
(Intercept)  55.4375     2.1196  26.155
Price2       -1.8375     0.4134  -4.444
Price3       -4.1000     0.4134  -9.917

Correlation of Fixed Effects:
       (Intr) Price2
Price2 -0.098
Price3 -0.098  0.500
```

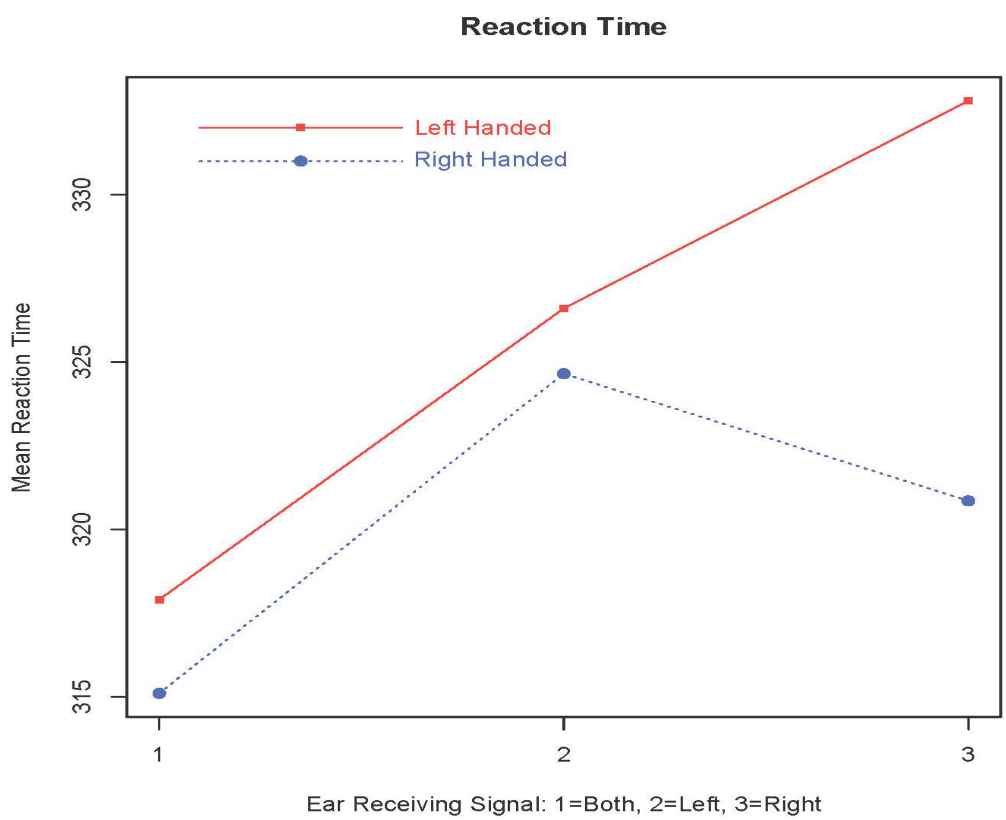
```
> anova(gfmixed)
Analysis of Variance Table
      Df Sum Sq Mean Sq F value
Price  2 67.481   33.74  49.346
> # No p-values
> Anova(gfmixed, test="F") # F test (from car package)
Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)

Response: Sales
           F Df Df.res       Pr(>F)
Price 49.346  2     14 0.0000004567 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> # Compare naive fixed effects model
> anova(lm(Sales ~ Price))
Analysis of Variance Table

Response: Sales
          Df Sum Sq Mean Sq F value Pr(>F)
Price      2  67.48  33.740  0.9388 0.4069
Residuals 21 754.76  35.941
```

Dichotic listening

Left-handed and right-handed subjects push a key when they hear their names over background noise. They are wearing stereo headphones. The signal comes in the left ear, the right ear, or both. There are 50 trials in each condition, presented in a different random order for each subject. The response variable is median reaction time in milliseconds. Each subject contributes 3 medians.

```
> rm(list=ls()); options(scipen=999) # To avoid scientific notation
> # Install packages if necessary. Only need to do this once.
> # install.packages("lme4")
> # install.packages("car")
> # Load packages -- do this every time
> library(lme4) # For lmer function
Loading required package: Matrix
> library(car)  # For F-tests, likelihood ratio and Wald chi-squared tests
> # Avoid the lmerTest package. It seems to have actual errors.
>
> # Read data into a data frame
> dichotic =
read.table("http://www.utstat.toronto.edu/brunner/data/legal/HandEar.data.txt")
> head(dichotic); attach(dichotic)

  subject handed   ear rtime
1       1   Left  Left   330
2       1   Left Right   327
3       1   Left  Both   303
4       2   Left  Left   294
5       2   Left Right   339
6       2   Left  Both   315
>
> # Sample sizes
> table(handed,ear)

       ear
handed  Both Left Right
  Left    20   20    20
  Right   20   20    20
>
> # Table of means
> TwoWay = tapply(rtime, INDEX = list(handed,ear), FUN=mean); TwoWay

       Both   Left  Right
Left  317.9 326.60 332.80
Right 315.1 324.65 320.85
>
>  # Add marginal means to table
> round(addmargins(TwoWay,margin=c(1,2),FUN=mean, quiet =TRUE),1)

       Both  Left Right  mean
Left  317.9 326.6 332.8 325.8
Right 315.1 324.6 320.9 320.2
mean  316.5 325.6 326.8 323.0
```

```
> TwoWay
       Both   Left  Right
Left  317.9 326.60 332.80
Right 315.1 324.65 320.85


> # Plot the means
> lhand = TwoWay[1,] # Row 1
> rhand = TwoWay[2,] # Row 2
> Ear = c(1:3,1:3)        # x points
> Means = c(lhand,rhand)  # y points
> # Invisible points at first, x axis points at 1,2,3; see help(plot)
> plot(Ear,Means,pch=" ",xaxp=c(1,3,2),
+     xlab="Ear Receiving Signal: 1=Both, 2=Left, 3=Right",
+     ylab="Mean Reaction Time")
> title("Reaction Time")
> points(1:3,lhand,col='red',pch=15) # Red squares
> points(1:3,rhand,col='blue',pch=19) # Blue circles
> lines(1:3,lhand,lty=1,col='red'); lines(1:3,rhand,lty=3,col='blue')
> # Annotate the plot
> x1 = c(1.1,1.6); y1 = c(332,332); lines(x1,y1,lty=1,col='red')
> points(1.35,332,col='red',pch=15)
> text(1.80,332,'Left Handed',col='red')
> x2 = c(1.1,1.6); y2 = c(331,331); lines(x2,y2,lty=3,col='blue')
> points(1.35,331,col='blue',pch=19)
> text(1.82,331,'Right Handed',col='blue')
>
```



```
> # Naive fixed effects analysis
> anova(lm(rtime ~ handed*ear))
Analysis of Variance Table

Response: rtime
            Df Sum Sq Mean Sq F value  Pr(>F)
handed       1    930  929.63  1.8556 0.17582
ear          2   2551 1275.41  2.5458 0.08286 .
handed:ear   2    615  307.41  0.6136 0.54317
Residuals  114  57113  500.99
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> # Repeated measures with a mixed model
> dichotic = lmer(rtime ~ handed*ear + (1 | subject))
> Anova(dichotic, test="F") # F tests (from car package)
Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)

Response: rtime
                F Df Df.res  Pr(>F)
handed     0.9706  1     38 0.33075
ear        4.6787  2     76 0.01213 *
handed:ear 1.1277  2     76 0.32914
---
Signif. Codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

>
> # Multiple comparisons: Which marginal means are different?
>
> # The combination variable HandEar will have 6 values
> HandEar = paste(handed,ear,sep='')
> table(HandEar) # Sample sizes

HandEar
  LeftBoth   LeftLeft  LeftRight  RightBoth  RightLeft RightRight
        20         20         20         20         20         20
>
> # Want a table of means in a similar format.
> ComboMeans = tapply(rtime, INDEX = list(HandEar), FUN=mean); ComboMeans
  LeftBoth   LeftLeft  LeftRight  RightBoth  RightLeft RightRight
    317.90     326.60     332.80     315.10     324.65     320.85
```

```
>
> # For a no-intercept model on a combination variable, the regression
> # coefficients are the treatment combination means.
>
> # Fit a no-intercept model
> ComboModel = lmer(rtime ~ 0 + HandEar + (1 | subject))
>
> # Contrast matrix for testing Ear, just as a check
> CM1 = rbind(c(1, -1,  0,  1, -1,  0), # Both - Left
+             c(0,  1, -1,  0,  1, -1)) # Left - Right
> colnames(CM1) = sort(unique(HandEar)) # Makes it easier to look at
> CM1
     LeftBoth LeftLeft LeftRight RightBoth RightLeft RightRight
[1,]        1       -1         0         1        -1          0
[2,]        0        1        -1         0         1         -1
> linearHypothesis(ComboModel,CM1,test="F") # Compare F = 4.6787
Linear hypothesis test

Hypothesis:
HandEarLeftBoth - HandEarLeftLeft  + HandEarRightBoth - HandEarRightLeft = 0
HandEarLeftLeft - HandEarLeftRight  + HandEarRightLeft - HandEarRightRight = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df      F  Pr(>F)
1     78
2     76  2 4.6787 0.01213 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> # Now 3 pairwise comparisons of marginal means for ear.
> # Look at the means again
> round(addmargins(TwoWay,margin=c(1,2),FUN=mean, quiet =TRUE),1)
       Both  Left Right  mean
Left  317.9 326.6 332.8 325.8
Right 315.1 324.6 320.9 320.2
mean  316.5 325.6 326.8 323.0
>
> bothVSleft = c(1, -1,  0,  1, -1,  0)
> linearHypothesis(ComboModel,bothVSleft,test="F")

Linear hypothesis test

Hypothesis:
HandEarLeftBoth - HandEarLeftLeft  + HandEarRightBoth - HandEarRightLeft = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df     F  Pr(>F)
1     77
2     76  1 6.109 0.01569 *
---
Signif. Codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> bothVSright = c(1,  0, -1,  1,  0, -1)
> linearHypothesis(ComboModel,bothVSright,test="F")
Linear hypothesis test

Hypothesis:
HandEarLeftBoth - HandEarLeftRight  + HandEarRightBoth - HandEarRightRight = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df      F   Pr(>F)   
1     77                      
2     76  1 7.8214 0.006536 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

> leftVSright = c(0,  1, -1,  0,  1, -1)
> linearHypothesis(ComboModel,leftVSright,test="F")
Linear hypothesis test

Hypothesis:
HandEarLeftLeft - HandEarLeftRight  + HandEarRightLeft - HandEarRightRight = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df      F Pr(>F)
1     77                 
2     76  1 0.1056  0.746
```

Faster reaction to a signal coming into both ears.

```
       Both  Left Right  mean
Left  317.9 326.6 332.8 325.8
Right 315.1 324.6 320.9 320.2
```

The Dichotic Listening data are balanced, and tests from `lmer` (with the `car` package) are the classical exact Fs. These do not exist for unbalanced designs. However, the F approximations for fixed effects are excellent.
