$c = 5$ MULTINOMIAL MARGINAL

Example: Set $n = 20$, $n_1 = 1$, $n_2 = 8$, $n_3 = 5$

n_1	n_2	n_3	n_4	$n_5 = 20 - n_1 - n_2 - n_3 - n_4$
1	8	5	0	6
1	8	5	1	5
1	8	5	2	4
1	8	5	3	3
1	8	5	4	2
1	8	5	5	1
1	8	5	6	0

n_4 can range from 0 to $n - n_1 - n_2 - n_3 = 6$

n_{c-1} " " " 0 " $n - \sum_{j=1}^{c-2} n_j = k$

n_c also ranges from 0 to k, because either of the last 2 categories could get all k

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Set $s = \sum_{j=1}^{c-2} n_j$ = sum of fixed entries

$s + n_{c-1} + n_c = n$

$\Leftrightarrow n_{c-1} + n_c = n - s = k$

$n_c = k - n_{c-1}$

n_{c-1} can go from 0 to k
could get none or 1 or ...
or get them all

Marginal Adding over n_{c-1} Recalling $n_c = k - n_{c-1}$

$$\sum_{n_{c-1}=0}^{k} \binom{n}{n_1\ n_2 \cdots n_{c-1}\ \ n_c} \pi_1^{n_1} \pi_2^{n_2} \cdots \pi_{c-1}^{n_{c-1}} \pi_c^{n_c}$$

$$= \sum_{n_{c-1}=0}^{k} \frac{n!}{n_1!\, n_2! \cdots n_{c-1}!\, (k-n_c)!} \pi_1^{n_1} \cdots \pi_{c-1}^{n_{c-1}} \pi_c^{k-n_c}$$

$$= \frac{n!}{n_1!\, n_2! \cdots n_{c-2}!} \pi_1^{n_1} \cdots \pi_{c-2}^{n_{c-2}} \frac{1}{k!} \sum_{n_{c-1}=0}^{k} \frac{k!}{n_{c-1}!\, (k-n_{c-1})!} \pi_{c-1}^{n_{c-1}} \pi_c^{k-n_c}$$

$$= \binom{n}{n_1\ n_2 \cdots n_{c-2}\ \ k} \pi_1^{n_1} \cdots \pi_{c-2}^{n_{c-2}} \underbrace{\sum_{n_{c-1}=0}^{k} \binom{k}{n_{c-1}} \pi_{c-1}^{n_{c-1}} \pi_c^{k-n_c}}_{\sum_{j=0}^{n} \binom{n}{j} a^j b^{n-j} = (a+b)^n \text{ binom thm.}}$$

$$= \binom{n}{n_1\ n_2 \cdots n_{c-2}\ (n_{c-1}+n_c)} \pi_1^{n_1} \pi_2^{n_2} \cdots \pi_{c-2}^{n_{c-2}} (\pi_{c-1}+\pi_c)^{n_{c-1}+n_c}$$

Just collapsed!