STA 312f22 Assignment Two[1]

These questions are practice for the quiz on Friday Sept. 30th, and are not to be handed in.

1. Customers arrive at a Tim Hortons according to a Poisson process with rate $\lambda = 30$ per hour. What is the probability that exactly 40 customers arrive during a one-hour period? The answer is a number. My answer is 0.01394346.

2. For years, brand awareness for Big Red chewing gum has been stuck at about 6%, meaning that about 6% of consumers who chew gum say they remember hearing about Big Red gum. The marketing department is planning an advertising campaign to increase brand awareness, in the hope that increased brand awareness will lead to increased sales. Once the campaign was running a few weeks, they interviewed a random sample of 200 gum chewers, and found that twenty had heard of Big Red.

   (a) State a reasonable model for these data.

   (b) Without any derivation, estimate the brand awareness for Big Red, in percent. Your answer is a number between zero and one hundred.

   (c) Give an approximate 95% confidence interval for the brand awareness in percent. Your answer is a set of two numbers. My upper confidence limit is 0.1416.

   (d) What is the null hypothesis corresponding to the *main question*, in symbols?

   (e) What is the critical value (or values) of the test statistic at $\alpha = 0.05$ for a 2-sided test? The answer is a number or a pair of numbers.

   (f) Calculate the test statistic Z_2 — see lecture notes. The formulas for Z_1 and Z_2 will be provided with the quiz if necessary. What is the value of the test statistic? Your answer is a number. Show some work. My answer is 1.88.

      i. Do you reject H_0 at $\alpha = 0.05$? Answer Yes or No.

      ii. Do the data provide adequate evidence against the null hypothesis?

      iii. In plain, non-statistical language, what do you conclude? Your answer is a statement about brand awareness.

   (g) Some clever person suggests that for this problem, the test based on Z_1 will always be bigger than Z_2 as long as $p > \pi_0$. Try it and see. What is the value of Z_1? Your answer is a number. Show some work.

      i. Do you reject H_0 at $\alpha = 0.05$? Answer Yes or No.

      ii. What is the critical value (or values) of the test statistic at $\alpha = 0.05$ for a 2-sided test?

      iii. Do the data provide adequate evidence against the null hypothesis?

      iv. In plain, non-statistical language, what do you conclude? Your answer is a statement about brand awareness.

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(h) But is Z_1 *always* bigger than Z_2 when $p > \pi_0$, as claimed? To answer this question, it is easier to base the test on Z^2 rather than Z. The null hypothesis is rejected when $|Z| > 1.96$, which occurs if and only if $Z^2 > 1.96^2 = \chi^2_{0.05(1)}$ – so it's the same test. Now find the value of p for which the denominator of Z_2^2 is greatest. Make a rough sketch of the function. Now you can answer the question: Is Z_1 *always* bigger than Z_2 when $p > \pi_0$?

3. Ten friends have a party right after graduating from university. At the time, none of them has ever been married. The party includes a visit by a fortune teller, who says "Five years from now, 3 of you will still be unmarried, 3 of you will be married for the first time, 2 will be divorced, one will be married for the second time, and one will be widowed."

   How many ways are there for this to happen? The answer is a number. Show some work. My answer is 50,400.

4. Students entering U of T have to choose a division: Humanities, Social Sciences, or Sciences.

   (a) Of the 25 students from a particular high school, how many ways are there for 8 to choose the Humanities, 14 to choose the Social Sciences and 3 to choose the Sciences? The answer is a number. My answer is 735,471,000, but you can just leave it in factorial form.

   (b) Of the 3 students from another high school, how many ways are there for 1 to choose the Humanities, 1 to choose the Social Sciences and 1 to choose the Sciences? The answer is a number. Show your work.

5. Please do problem 1.6 from the text. For (b), I get 10 possibilities. My answer to (c) is 3/16.

6. A fair die is tossed 8 times. What is the probability of observing the numbers 3 and 4 twice each, and the others once each? The answer is a number. My answer is around 0.006.

7. A box contains 5 red, 3 white and two blue marbles. A sample of six marbles is drawn with replacement. Find the probability that

   (a) 3 are red, 2 are white and one is blue. My answer is 0.135.

   (b) 2 are red, 3 are white and 1 is blue. My answer is 0.0810.

   (c) 2 of each colour appear. My answer is 0.0810.

8. Let $\mathbf{Y}_1, \ldots, \mathbf{Y}_n$ be a random sample from a $M\left(1, (\pi_1, \ldots, \pi_c)\right)$ distribution. Show why the likelihood function is written $\ell(\boldsymbol{\pi}) = \pi_1^{n_1}\pi_2^{n_2}\cdots\pi_c^{n_c}$.

9. Let $\mathbf{Y}_1, \ldots, \mathbf{Y}_n$ be a random sample from a $M\left(1, (\pi_1, \pi_2, \pi_3)\right)$ distribution. Find the maximum likelihood estimator of (π_1, π_2, π_3). Show *all* your work.
