# STA 312f12 Assignment Four[1]

Please bring your R printouts from the last question to the quiz.The other questions are practice for the quiz on Friday Oct. 12th, and are not to be handed in – even when questions in the text ask for a $p$-value.

On the quiz, you will be given a copy of the formula sheet, so please use the formula sheet while doing your homework, and let me know if there are problems. There is a link to the formula sheet from the course home page, in case the one in this document does not work.

1. In the text starting on p. 55, please do problems 2.3-2.7.

2. Do 2.12 just parts $a$ and $b$.

3. Do 2.14. What homework problem from Assignment 3 shows you how to test the difference between those 2 percentages? What one number is missing?

4. Do 2.16, parts $a$ through $c$. Look at how many smokers there were in those days! Part $b$ is asking whether the study is prospective, retrospective or cross-sectional. Now suppose that in this population the overall probability of lung cancer is one percent. Using this information and the data in the table,

    (a) Estimate the probability of lung cancer given that the person is a smoker.

    (b) Estimate the relative risk of lung cancer for smokers (compared to non-smokers. Your answer is a number.

5. Do 2.17, 2.18 and 2.21. For each question, state whether the study is prospective, retrospective or cross-sectional.

6. Using the notation in this table and assuming a cross-sectional study,

|  | $Y = 1$ | $Y = 2$ | Total |
|---|---|---|---|
| $X = 1$ | $\pi_{11}$ | $\pi_{12}$ | $\pi_{11} + \pi_{12}$ |
| $X = 2$ | $\pi_{21}$ | $\pi_{22}$ | $\pi_{21} + \pi_{22}$ |
| Total | $\pi_{11} + \pi_{21}$ | $\pi_{12} + \pi_{22}$ |  |

    (a) What are the odds of $Y = 1$ given $X = 1$? Just write the answer down.

    (b) What are the odds of $Y = 1$ given $X = 2$? Just write the answer down.

    (c) What are the odds of $Y = 1$ given $X = 1$ *divided by* the odds of $Y = 1$ given $X = 2$? Simplify.

    (d) Show that $P(Y = 1|X = 1) = P(Y = 1|X = 2)$ if and only if $\theta = 1$.

    (e) Show that $P(X = 1|Y = 1) = P(X = 1|Y = 2)$ if and only if $\theta = 1$.

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(f) The two-by-two table is a matrix. Show that the odds ratio equals one if and only if the determinant of the matrix is zero.

7. For this question, you might want to use the notation shown in the following table. Assume it's a cross-sectional study.

| $x$ | $a - x$ | $a$ |
|---|---|---|
| $b - x$ | $1 - a - b + x$ | $1 - a$ |
| $b$ | $1 - b$ | $1$ |

(a) How many free parameters are there under the null hypothesis of independence? In the notation of the table, what are they?

(b) Write the likelihood function, restricted by the null hypothesis of independence. Use the usual symbols $n_1, n_2, n_3, n_4$ for the cell frequencies.

(c) Find the MLE of the free parameters of the restricted model. Show your work.

(d) Write down the entire restricted MLE in a $2 \times 2$ table.

(e) Based on your work, give a formula for $\widehat{\mu}_{ij}$. Simplify.

(f) Switching back to the usual $\pi_{ij}$ notation of the table in Question 6, find $\pi_{ij}(M)$ for $i = 1, 2$ and $j = 1, 2$. Use the Law of Large Numbers, which basically says $p \to \pi$. Show your work

(g) Give a formula for the non-centrality parameter $\lambda$ for the Pearson $X^2$ test of independence, applied to a $2 \times 2$ table.

8. Assume the following true probabilities:

| | $Y = 1$ | $Y = 2$ |
|---|---|---|
| $X = 1$ | $\pi_{11} = 0.3$ | $\pi_{12} = 0.2$ |
| $X = 2$ | $\pi_{21} = 0.1$ | $\pi_{22} = 0.4$ |

What sample size is required for the Pearson $X^2$ test of independence to have a Power of 0.80? The answer is a number. *Bring your R printout to the quiz.* Do not write anything on it in advance except possibly your name and student numbber.