STA 305 Formulas

$Var(Y) = E\{(Y-\mu_y)^2\}$

$Cov(Y,T) = E\{(Y-\mu_y)(T-\mu_t)\}$

$cov(\mathbf{Y}) = E\left\{(\mathbf{Y}-\boldsymbol{\mu}_y)(\mathbf{Y}-\boldsymbol{\mu}_y)'\right\}$

$C(\mathbf{Y},\mathbf{T}) = E\left\{(\mathbf{Y}-\boldsymbol{\mu}_y)(\mathbf{T}-\boldsymbol{\mu}_t)'\right\}$

If $W = W_1 + W_2$ with W_1 and W_2 independent, $W \sim \chi^2(\nu_1+\nu_2)$, $W_2 \sim \chi^2(\nu_2)$ then $W_1 \sim \chi^2(\nu_1)$

$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$

$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1,\nu_2)$

For the multivariate normal distribution *only*, zero covariance implies independence.

If $\mathbf{Y} \sim N_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$, then $\mathbf{AY} \sim N_q(\mathbf{A}\boldsymbol{\mu},\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}')$, and $W = (\mathbf{Y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{Y}-\boldsymbol{\mu}) \sim \chi^2(p)$

$Y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_{p-1}x_{i,p-1} + \epsilon_i$

$\epsilon_1, \ldots, \epsilon_n$ independent $N(0,\sigma^2)$

$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$

$\boldsymbol{\epsilon} \sim N_n(\mathbf{0},\sigma^2\mathbf{I}_n)$

$\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$

$\widehat{\mathbf{Y}} = \mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{HY}$, where $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$

$\sum_{i=1}^n (Y_i-\overline{Y})^2 = \sum_{i=1}^n (Y_i-\widehat{Y}_i)^2 + \sum_{i=1}^n (\widehat{Y}_i-\overline{Y})^2$

$SST = SSE + SSR$ and $R^2 = \frac{SSR}{SST}$

$\widehat{\boldsymbol{\epsilon}} = \mathbf{Y} - \widehat{\mathbf{Y}}$

$\widehat{\boldsymbol{\beta}} \sim N_p\left(\boldsymbol{\beta},\sigma^2(\mathbf{X}'\mathbf{X})^{-1}\right)$

$\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\epsilon}}$ are independent under normality.

$SSE/\sigma^2 = \hat{\boldsymbol{\epsilon}}'\hat{\boldsymbol{\epsilon}}/\sigma^2 \sim \chi^2(n-p)$

$T = \frac{\mathbf{a}'\widehat{\boldsymbol{\beta}} - \mathbf{a}'\boldsymbol{\beta}}{\sqrt{MSE\,\mathbf{a}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}}} \sim t(n-p)$

$F^* = \frac{(\mathbf{C}\widehat{\boldsymbol{\beta}}-\mathbf{t})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\boldsymbol{\beta}}-\mathbf{t})}{q\,MSE} \sim F(q,n-p,\lambda)$

$F^* = \frac{SSR - SSR(reduced)}{q\,MSE} \sim F(q,n-p,\lambda)$

where $MSE = \frac{SSE}{n-p}$

$\lambda = \frac{(\mathbf{C}\boldsymbol{\beta}-\mathbf{t})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\boldsymbol{\beta}-\mathbf{t})}{\sigma^2}$

Simple random sample of n units from N without replacement. $Z_i = 1$ if unit i is chosen, zero otherwise.

$E(Z_i) = P(Z_i = 1) = \frac{n}{N}$

$\overline{y}_u = \frac{1}{N}\sum_{i=1}^N y_i$

$\overline{y} = \frac{1}{n}\sum_{i=1}^N Z_i y_i$

$S^2 = \frac{1}{N-1}\sum_{i=1}^N (y_i - \overline{y}_u)^2$

$c = a_1\mu_1 + a_2\mu_2 + \cdots + a_p\mu_p$

$\widehat{c} = a_1\overline{Y}_1 + a_2\overline{Y}_2 + \cdots + a_p\overline{Y}_p$

$Pr\left\{\cup_{j=1}^k A_j\right\} \leq \sum_{j=1}^k Pr\{A_j\}$

Reject H_0 with a Scheffé test if $F_2 > \frac{q}{s} f_\alpha(q,n-p)$

$n = \frac{\sigma^2\, z_{\alpha/2}^2 \sum_{j=1}^p \frac{a_j^2}{f_j}}{m^2}$

$1-\alpha$	0.80	0.90	0.95	0.99
$z_{\alpha/2}$	1.28	1.64	1.96	2.58