STA 302 Summer 2001 Quiz Five

1. (4 points) For the Brand Preference data, obtain a 95% prediction interval for the mean of three new observations that will all have $X_{h1} = 5$ and $X_{h2} = 4$. The critical value you need is $t(.975; 13) = 2.16$.

2. In a market research study on shoe sales, Y represents sales, x_1 is price, and s_1, s_2 and s_3 are dummy variables for season, defined as follows: $s_1 = 1$ if winter and zero otherwise, $s_2 = 1$ if spring and zero otherwise, and $s_3 = 1$ if fall and zero otherwise. The model (supressing $i = 1, \ldots, n$) is

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 s_1 + \beta_3 s_2 + \beta_4 s_3 + \beta_5 x_1 s_1 + \beta_6 x_1 s_2 + \beta_7 x_1 s_3 + \epsilon$$

   (a) (2 points) In this question, if the *symbols* s_1, s_2 and s_3 appear in your answer, it is wrong.
      i. What is $E[Y]$ for Winter?
      ii. What is $E[Y]$ for Spring?
      iii. What is $E[Y]$ for Fall?
      iv. What is $E[Y]$ for Summer?

   (b) (2 points) The Marketing Manager believes that the slope of the line relating price to expected sales should be the same in the Winter, Spring and Fall. She has no firm opinion about whether it might be different in Summer. You are asked to formally test the Manager's idea. Give just the $\mathbf{C}$ matrix in $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$.

   (c) (2 points) Suppose you wanted to test whether the regression relations were identical in all four seasons (equal intercepts as well as slopes). Give just the $\mathbf{C}$ matrix in $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$.

```
/* q5.sas */
options linesize=79 pagesize=500;
title 'Quiz 5 Job';

data portrait;
     infile 'brandpref.dat';
     input liking moist sweet;

proc glm;
     model liking = moist sweet;
     estimate 'Xh' intercept 1 moist 5 sweet 4;

/*
```

The GLM Procedure

Number of observations 16

Quiz 5 Job

The GLM Procedure

Dependent Variable: liking

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	1872.700000	936.350000	129.08	<.0001
Error	13	94.300000	7.253846		
Corrected Total	15	1967.000000			

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Xh	77.2750000	1.12668677	68.59	<.0001

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Intercept	37.65000000	2.99610324	12.57	<.0001
moist	4.42500000	0.30111971	14.70	<.0001
sweet	4.37500000	0.67332413	6.50	<.0001

Jerry's Answers to Quiz 5

① $\hat{\sigma}^2\{\text{predmean}\} = \frac{MSE}{2} + \hat{\sigma}^2\{\hat{\gamma}_1\} = \frac{7.25}{2} + 1.1267^2 = 4.89$,

and $\hat{\gamma}_h \pm t(1-\alpha/2; n-p)\,\hat{\sigma}\{\text{predmean}\} = 77.275 \pm (2.16)(2.21)$

$= 77.275 \pm 4.774 = \underline{(72.501, 82.049)}$

② a)

SEASON	E(Y)
WINTER	$(\beta_0+\beta_2) + (\beta_1+\beta_5)X_1$
SPRING	$(\beta_0+\beta_3) + (\beta_1+\beta_6)X_1$
FALL	$(\beta_0+\beta_4) + (\beta_1+\beta_7)X_1$
SUMMER	$\beta_0 \quad \beta_1 \quad X_1$

b) $\underset{\sim}{C} = \begin{bmatrix} 0&0&0&0&0&1&-1&0 \\ 0&0&0&0&0&0&1&-1 \end{bmatrix}$

c) $\underset{\sim}{C} = \begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}$