STA 302 Summer 2001 Quiz Two

1. (5 Points) For the simple regression model $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$,
   (a) What is the expected value of Y_i? Showing work on this part is optional.
   (b) What is the expected value of $\overline{Y} = \frac{1}{n}\sum_{i=1}^n Y_i$? Show your work.
   (c) Use the work you have just done to show that b_1 is unbiased. Show your work. You do not need to prove

$$b_1 = \frac{\sum_{i=1}^n (x_i - \overline{x})(Y_i - \overline{Y})}{\sum_{i=1}^n (x_i - \overline{x})^2}$$

2. The printout below is based on the Grade Point Average data from your homework, in which an attempt was made to predict first-year university grade point average from score on a test.
   (a) (1 Point) When the test score increases by *two* points, estimated GPA increases by how much? The answer is a number.
   (b) (1 Point) What proportion of the variation in GPA is explained by test score? The answer is a number
   (c) (3 Points) Your boss, who has a short temper and has never had a statistics course, asks you to interpret the t-test for the intercept on the printout. Reply, *in everday language*. Remember, using terms like "null hypothesis" could get you fired, and will certainly lose you marks on this question, even if your answer is otherwise correct.

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Dependent Variable: gpa

                         Analysis of Variance

                                 Sum of           Mean
Source                   DF     Squares         Square    F Value    Pr > F

Model                     1     6.43373        6.43373      34.00    <.0001
Error                    18     3.40627        0.18924
Corrected Total          19     9.84000


          Root MSE              0.43501    R-Square     0.6538
          Dependent Mean        2.50000    Adj R-Sq     0.6346
          Coeff Var            17.40057

                        Parameter Estimates

                     Parameter       Standard
Variable      DF      Estimate          Error    t Value    Pr > |t|

Intercept      1      -1.69956        0.72678      -2.34      0.0311
test           1       0.83991        0.14405       5.83      <.0001
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Jerry's Answers to Quiz 2

1a) $E(Y_i) = \beta_0 + \beta_1 x_i$

b) $E(\bar{Y}) = E\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(Y_i)$

$$= \frac{1}{n}\sum_{i=1}^{n}(\beta_0 + \beta_1 x_i) = \frac{1}{n}\left[n\beta_0 + \beta_1 \sum_{i=1}^{n} x_i\right] = \boxed{\beta_0 + \beta_1 \bar{x}}$$

c) $$E(b_1) = E\left[\frac{\sum_{i=1}^{n}(x_i - \bar{x})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right]$$

$$= \frac{\sum_{i=1}^{n}(x_i - \bar{x})E[(Y_i - \bar{Y})]}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(E(Y_i) - E(\bar{Y}))}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$= \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\beta_0 + \beta_1 x_i - \beta_0 - \beta_1 \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \beta_1 \frac{\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$= \beta_1$ UNBIASED

2a) $2 * 0.83991 = \boxed{1.68}$

b) $\boxed{.6538}$

c) It is meaningless. The intercept is estimated GPA for students who got zero on the test, and likely there were no such students in the data.