This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Quiz 6

Due: Thursday October 29, 2020 6:30 PM (EDT)

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Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (7 points)

Let $\mathbf{x}_1$ be a p-dimensional random vector with moment-generating function $M_{\mathbf{x}_1}(\mathbf{t})$, and let $\mathbf{x}_2$ be another p-dimensional random vector, with moment-generating function $M_{\mathbf{x}_2}(\mathbf{t})$. Show that if $\mathbf{x}_1$ and $\mathbf{x}_2$ are independent, $M_{\mathbf{x}_1+\mathbf{x}_2}(\mathbf{t}) = M_{\mathbf{x}_1}(\mathbf{t})M_{\mathbf{x}_2}(\mathbf{t})$. In your proof, assume that $\mathbf{x}_1$ and $\mathbf{x}_2$ are continuous (so they have joint density functions), and integrate. Clearly indicate where you use the independence of $\mathbf{x}_1$ and $\mathbf{x}_2$.

Q2 (3 points)

Let $\mathbf{x}_1 \sim N_p(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$ and $\mathbf{x}_2 \sim N_p(\boldsymbol{\mu}_2, \boldsymbol{\Sigma}_2)$ be independent multivariate normal random vectors. Using moment-generating functions, find the distribution of $\mathbf{x}_1 + \mathbf{x}_2$.

Quiz 6

(1) $M_{x_1+x_2}(t) = E\left(e^{t'(x_1+x_2)}\right)$

$$= \int\int e^{t'(x_1+x_2)} \underbrace{f_{x_1}(x_1)\, f_{x_2}(x_2)}_{ind}\, dx_1\, dx_2$$

$$= \int\int e^{t'x_1+t'x_2} f_{x_1}(x_1)\, f_{x_2}(x_2)\, dx_1\, dx_2$$

$$= \int\int e^{t'x_1} e^{t'x_2} f_{x_1}(x_1)\, f_{x_2}(x_2)\, dx_1\, dx_2$$

$$= \int e^{t'x_2} f_{x_2}(x_2) \underbrace{\int e^{t'x_1} f_{x_1}(x_1)\, dx_1}_{M_{x_1}(t)}\, dx_2$$

$$= M_{x_1}(t) \int e^{t'x_2} f_{x_2}(x_2)\, dx_2$$

$$= M_{x_1}(t)\, M_{x_2}(t)$$

(2) By Question 1,

$$M_{x_1+x_2}(t) = M_{x_1}(t)\, M_{x_2}(t)$$

$$= e^{t'\mu_1 + \frac{1}{2} t' \Sigma_1 t}\, e^{t'\mu_2 + \frac{1}{2} t' \Sigma_2 t}$$

$$= e^{t'\mu_1 + \frac{1}{2} t' \Sigma_1 t + t'\mu_2 + \frac{1}{2} t' \Sigma_2 t}$$

$$= e^{t'(\mu_1+\mu_2) + \frac{1}{2} t'(\Sigma_1+\Sigma_2) t}$$

MGF of $N_p(\mu_1+\mu_2,\ \Sigma_1+\Sigma_2)$