## Quiz 5

## Submit your assignment

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

## Q1 (6 points)

For $i=1, \ldots, n$, let $y_{i}=\beta_{1} x_{i, 1}+\beta_{2} x_{i, 2}+\epsilon_{i}$, where the $x_{i, j}$ are known constants and $\epsilon_{1}, \ldots, \epsilon_{n}$ are independent with expected value 0 and variance $\sigma^{2}$. We seek to estimate the following linear combination of $\beta$ values: $\ell_{1} \beta_{1}+\ell_{2} \beta_{2}$, where $\ell_{1}$ and $\ell_{2}$ are known constants. The estimator will be the following linear combination of the $y$ values: $L=\sum_{i=1}^{n} c_{i} y_{i}$. This is the setting of the Gauss-Markov Theorem.

Suppose that $L$ is an unbiased estimator; that is, suppose $E(L)=\ell_{1} \beta_{1}+\ell_{2} \beta_{2}$ for all real $\beta_{1}$ and $\beta_{2}$. Prove that $\ell_{1}=\sum_{i=1}^{n} c_{i} x_{i, 1}$ and $\ell_{2}=\sum_{i=1}^{n} c_{i} x_{i, 2}$.

## Q2 (4 points)

For this question, you will upload your complete answer to Question 16 of Assignment 5, based on the SAT data. Part (c) asked you to calculate and display the mean of the $\widehat{y}_{i}$. The answer is a number. Circle the number on your R input/output. If for some technical reason you are unable to circle the number, you may write it on a separate sheet and unload that as well. You cannot get any marks on this question without your complete answer to Question 16.

Quin 5
(1)

$$
\begin{aligned}
& E(L)=E\left(\sum_{i=1}^{n} c_{i} y_{i}\right)=\sum_{i=1}^{n} c_{i} E\left(y_{1}\right) \\
& =\sum_{i=1}^{n} c_{i}\left(\beta_{1} x_{i 1}+\beta_{2} x_{i 2}\right)=\beta_{1} \sum_{i=1}^{n} c_{i} x_{i 1}+\beta_{2} \sum_{i=1}^{n} c_{i} x_{i 2} \\
& =l_{1} \beta_{1}+l_{2} \beta_{2} \text { for all rall } \beta_{1} \not \& \beta_{2} \text {. In }
\end{aligned}
$$

particular, it is true for $\beta_{1}=1 \not \beta_{2}=0$, So $\sum_{i=1}^{n} c_{i} x_{i}=l_{1}$. It is cleo bore for $\beta_{1}=0 \neq \beta_{2}=1$, so $\sum_{i=1}^{n} c_{i} x_{i 2}=l_{2}$. done
(2)

No marks of the answer is based on lm . Matrix operations wed specified.

At best part marks if the answer to $Q 16$ is incomplete.

