This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Quiz 4

Due: Thursday October 8, 2020 6:30 PM (EDT)

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (5 points)

Independently for $i = 1, \ldots, n$, let $y_i = \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i$. Notice that there is no intercept, and the second x variable is x_i^2, not $x_{i,2}$. The regression coefficients β_1 and β_2 are unknown constants, $x_1, \ldots, x_n$ are known, observable constants, and $\epsilon_1, \ldots, \epsilon_n$ are independent random variables with expected value zero. Start deriving the least squares estimates of β_1 and β_2 by minimizing the sum of squared differences between the y_i and their expected values. You don't have to finish the job. Stop when you have the normal equations. **Circle your final answer**.

Q2 (5 points)

Assume the general linear regression model in matrix form. Only one of the following statements is correct. Choose the correct statement and prove it. You may use anything on the formula sheet without proof. Scratch out anything you don't want marked.

- $\widehat{\epsilon} = (\mathbf{I} - \mathbf{H})\widehat{\mathbf{y}}$.
- $\widehat{\mathbf{y}} = (\mathbf{I} - \mathbf{H})\widehat{\epsilon}$.
- $\widehat{\epsilon} = (\mathbf{I} - \mathbf{H})\epsilon$.

Quiz 4

(1) Minimize $Q(\beta_1, \beta_2) = \sum_{i=1}^{n} (y_i - \beta_1 x_i - \beta_2 x_i^2)^2$

$$\frac{\partial Q}{\partial \beta_1} = \sum_{i=1}^{n} 2(y_i - \beta_1 x_i - \beta_2 x_i^2)(-x_i)$$

$$= -2 \sum_{i=1}^{n} (x_i y_i - \beta_1 x_i^2 - \beta_2 x_i^3)$$

$$= -2 \left(\sum_{i=1}^{n} x_i y_i - \beta_1 \sum_{i=1}^{n} x_i^2 - \beta_2 \sum_{i=1}^{n} x_i^3 \right) \overset{set}{=} 0$$

$$\Longrightarrow \sum_{i=1}^{n} x_i y_i = \beta_1 \sum_{i=1}^{n} x_i^2 + \beta_2 \sum_{i=1}^{n} x_i^3$$

$$\frac{\partial Q}{\partial \beta_2} = \sum_{i=1}^{n} 2(y_i - \beta_1 x_i - \beta_2 x_i^2)(-x_i^2)$$

$$= -2 \sum_{i=1}^{n} (x_i^2 y_i - \beta_1 x_i^3 - \beta_2 x_i^4)$$

$$= -2 \left(\sum_{i=1}^{n} x_i^2 y_i - \beta_1 \sum_{i=1}^{n} x_i^3 - \beta_2 \sum_{i=1}^{n} x_i^4 \right) \overset{set}{=} 0$$

$$\Longrightarrow \sum_{i=1}^{n} x_i^2 y_i = \beta_1 \sum_{i=1}^{n} x_i^3 + \beta_2 \sum_{i=1}^{n} x_i^4$$

And the normal equations are

$$\left(\sum_{i=1}^{n} x_i^2\right)\beta_1 + \left(\sum_{i=1}^{n} x_i^3\right)\beta_2 = \sum_{i=1}^{n} x_i y_i$$

$$\left(\sum_{i=1}^{n} x_i^3\right)\beta_1 + \left(\sum_{i=1}^{n} x_i^4\right)\beta_2 = \sum_{i=1}^{n} x_i^2 y_i$$

(2) Show $\hat{\varepsilon} = (I-H)\varepsilon$. From the formula sheet,

$$\hat{\varepsilon} = (I-H)y = (I-H)(X\beta+\varepsilon)$$

$$= (I-H)X\beta + (I-H)\varepsilon$$

$$= X\beta - HX\beta \quad + (I-H)\varepsilon$$

$$= X\beta - X\underbrace{(X'X)^{-1}X'X}_{I}\beta \quad + (I-H)\varepsilon$$

$$= X\beta - X\beta + (I-H)\varepsilon$$

$$= (I-H)\varepsilon \quad \checkmark$$