Quiz 10

Due: Thursday November 26, 2020 6:30 PM (EST)

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (7 points)

Suppose you have data from a regression with a sample of size n, and now you are about to sample one more observation independently from the same population: $y_0 = \mathbf{x}_0'\boldsymbol{\beta} + \epsilon_0$, where $\epsilon_0 \sim N(0, \sigma^2)$. For some reason you want a prediction interval for ϵ_0, even though you will never be able to observe it.

a. Obtain the appropriate t-distributed random variable. Make sure to state why the numerator and denominator are independent, and give the degrees of freedom.

b. Derive the $(1-\alpha)\times 100$ % prediction interval. Show your work. Use $t_{\alpha/2}$ to denote the critical value. **Circle your final answer.**

Q2 (3 points)

For the Puromycin data, a residual plot pointed to the need for a quadratic term in the regression equation. Please upload the residual plot and the code used to produce it. Your code should not do anything unnecessary. For example, you don't need "summary." I did it in two lines.

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(1) (a) $\varepsilon_0 \sim N(0, \sigma^2)$, so $Z = \frac{\varepsilon_0}{\sigma} \sim N(0,1)$

$$t = \frac{\varepsilon_0/\sigma}{\sqrt{\frac{SSE}{\sigma^2}/(n-k-1)}} = \frac{\varepsilon_0}{\sqrt{MSE}} \sim t(n-k-1)$$

Numerator and denominator are independent because SSE is based on $y_1, \ldots, y_n$ which are independent of ε_0.

(b) $$1-\alpha = P\left\{-t_{\alpha/2} < \frac{\varepsilon_0}{\sqrt{MSE}} < t_{\alpha/2}\right\}$$

$$= P\left\{-t_{\alpha/2}\sqrt{MSE} < \varepsilon_0 < t_{\alpha/2}\sqrt{MSE}\right\}$$

the prediction interval is

$$\pm\, t_{\alpha/2}\sqrt{MSE}$$

2.

R for Quiz 10

```
> model = lm(rate ~ conc+state, data=Puromycin)
> plot(Puromycin$conc,residuals(model),xlab='Concentration',ylab='Residual')
```