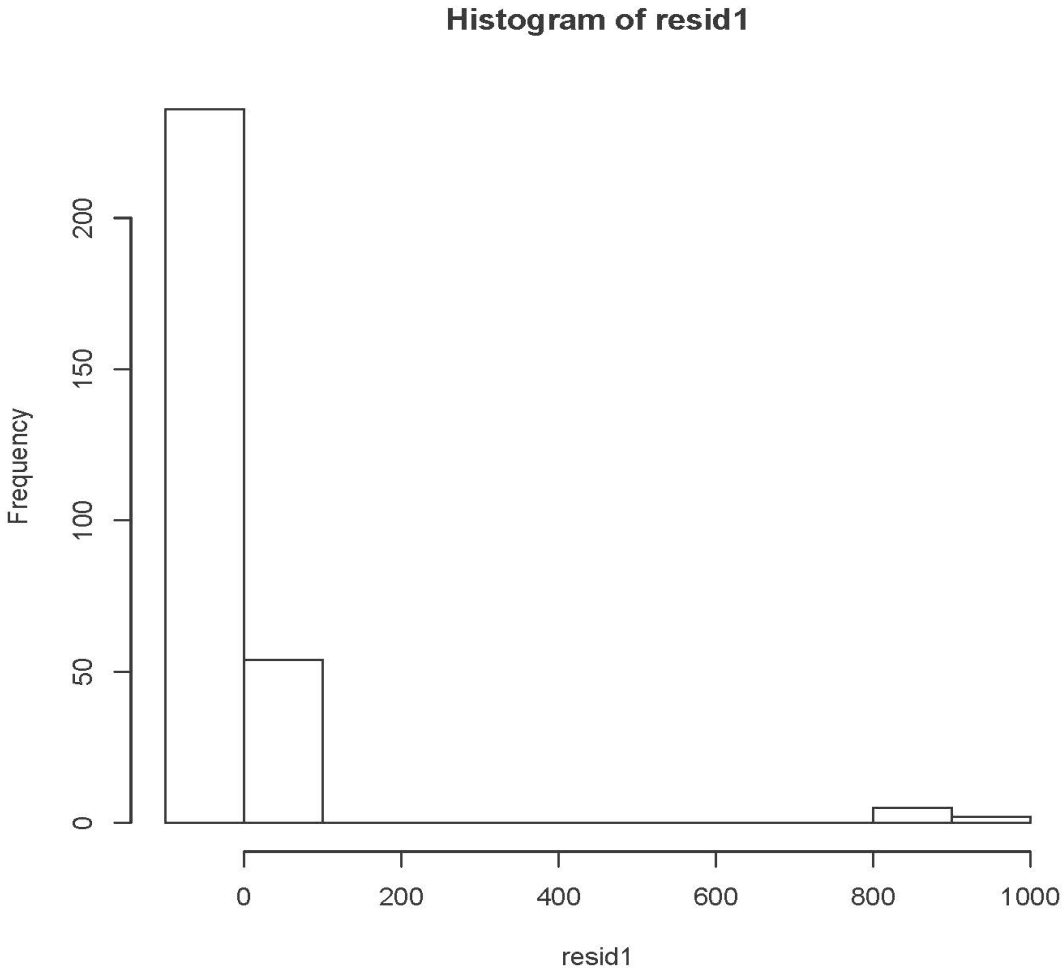
More Diagnostics With R*

```
> rm(list=ls())
> math =
read.table("http://www.utstat.utoronto.ca/~brunner/data/legal/math1.data.txt")

> head(math); summary(math)
    course precalc calc hsgpa hscalc hsengl ucalc frstlang sex
1 Mainstrm       2    0  78.0     65     80    39  English   F
2 Mainstrm       6    2  66.0     54     75    57  English   F
3 Mainstrm       4    4  80.2     77     70    62  English   M
4 Mainstrm       5    2  81.7     80     67    76  English   F
5 Mainstrm       4    4  86.8     87     80    86  English   M
6 Mainstrm       3    1  76.7     53     75    60  English   M

      course       precalc          calc            hsgpa
 Catch-up: 59   Min.   :0.000   Min.   : 0.000   Min.   : 0.00
 Elite   : 39   1st Qu.:3.000   1st Qu.: 1.000   1st Qu.:74.50
 Mainstrm:373   Median :4.000   Median : 3.000   Median :78.00
 NA's    :108   Mean   :4.402   Mean   : 3.319   Mean   :75.23
                3rd Qu.:6.000   3rd Qu.: 5.000   3rd Qu.:82.25
                Max.   :9.000   Max.   :11.000   Max.   :97.30
                NA's   :99      NA's   :99       NA's   :88
     hscalc          hsengl          ucalc          frstlang     sex
 Min.   :  0.00   Min.   : 0.00   Min.   :  0.0   English:402   F   :266
 1st Qu.: 65.00   1st Qu.:70.00   1st Qu.: 50.0   French :  5   M   :285
 Median : 75.00   Median :76.00   Median : 60.0   Other  :144   NA's: 28
 Mean   : 97.32   Mean   :74.44   Mean   :118.6   NA's   : 28
 3rd Qu.: 85.00   3rd Qu.:82.00   3rd Qu.: 75.0
 Max.   :999.00   Max.   :96.00   Max.   :999.0
 NA's   :88       NA's   :90      NA's   :157
>
> # Make a new data frame without the NAs. First, how many rows and columns?
> dim(math)
[1] 579   9
>
> Math = na.omit(math); dim(Math)
[1] 321   9
> summary(Math)
      course       precalc          calc            hsgpa
 Catch-up: 14   Min.   :1.000   Min.   : 0.000   Min.   : 0.00
 Elite   : 28   1st Qu.:4.000   1st Qu.: 2.000   1st Qu.:75.80
 Mainstrm:279   Median :5.000   Median : 4.000   Median :79.70
                Mean   :4.801   Mean   : 3.894   Mean   :77.42
                3rd Qu.:6.000   3rd Qu.: 6.000   3rd Qu.:83.80
                Max.   :9.000   Max.   :11.000   Max.   :96.20
     hscalc          hsengl          ucalc          frstlang   sex
 Min.   :  0.00   Min.   : 0.00   Min.   :  1.0   English:251   F:162
 1st Qu.: 70.00   1st Qu.:71.00   1st Qu.: 50.0   French :  1   M:159
 Median : 80.00   Median :77.00   Median : 61.0   Other  : 69
 Mean   : 78.13   Mean   :75.52   Mean   :103.5
 3rd Qu.: 86.00   3rd Qu.:83.00   3rd Qu.: 75.0
 Max.   :999.00   Max.   :96.00   Max.   :999.0
>
> # The following are impossible:
>     # hscalc = 999
>     # hscalc = 0
>     # hsengl = 0
>     # ucalc = 999
> # Make them NA and reduce the data set again.
>
```

```
> Math$hscalc[Math$hscalc==999] = NA
> Math$hscalc[Math$hscalc==0] = NA
> Math$hsengl[Math$hsengl==0] = NA
> Math$ucalc[Math$ucalc==999] = NA
>
> Math = na.omit(Math); dim(Math)
[1] 297   9
>
> # Also make that one Francophone Other
> Math$frstlang[Math$frstlang=='French'] = 'Other'
> attach(Math)
>
> # One more problem
> contrasts(frstlang) # Built-in dummy variable coding
        French Other
English      0     0
French       1     0
Other        0     1
> frstlang = factor(frstlang); contrasts(frstlang)
        Other
English     0
Other       1
>
> # Fit a model
> fullmodel1  = lm(ucalc ~ course + precalc + calc + hsgpa + hscalc + hsengl +
frstlang + sex)
> summary(fullmodel1)

Call:
lm(formula = ucalc ~ course + precalc + calc + hsgpa + hscalc +
    hsengl + frstlang + sex)

Residuals:
   Min     1Q Median     3Q    Max
-92.73 -36.83 -20.23  -4.48 906.13

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -26.8798   101.9621  -0.264    0.792
courseElite   -8.8504    53.8622  -0.164    0.870
courseMainstrm 22.2461   45.9467   0.484    0.629
precalc       10.4684     5.8985   1.775    0.077 .
calc           1.1838     3.9823   0.297    0.766
hsgpa          0.5359     0.9844   0.544    0.587
hscalc        -0.5950     0.9069  -0.656    0.512
hsengl         0.4007     1.1215   0.357    0.721
frstlangOther 22.2006    21.8397   1.017    0.310
sexM           7.1425    17.8935   0.399    0.690
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 143.9 on 287 degrees of freedom
Multiple R-squared:  0.02603, Adjusted R-squared:  -0.004512
F-statistic: 0.8523 on 9 and 287 DF,  p-value: 0.5686

>
> # Those residuals are really strange.
> resid1 = fullmodel1$residuals
> hist(resid1)
```




```
>
> # Track down those huge residuals
> n = length(course); n; id = 1:n
[1] 297
> crazy = id[resid1>700]; length(crazy)
[1] 7
> crazy
[1]  36  72  80  94 174 176 188
> Math[crazy,] # Just those 7 rows, all the columns
      course precalc calc hsgpa hscalc hsengl ucalc frstlang sex
63  Mainstrm       6    4  77.2     80     70   998  English   M
140 Mainstrm       4    4  81.0     65     83   998    Other   F
154 Mainstrm       7    5  85.3     89     74   998  English   M
183 Mainstrm       4    0  73.8     63     78   998    Other   M
343 Mainstrm       8    5  76.0     77     77   998  English   F
347 Mainstrm       6    8  92.7     98     84   998  English   F
365 Mainstrm       5    5  74.7     57     78   998  English   M
```

```
> sort(ucalc)
  [1]   1   2  12  13  16  17  17  18  19  19  20  22  26  26  28  28  28  30
 [19]  31  31  31  32  32  33  34  34  35  35  35  36  37  37  39  39  39  39
 [37]  39  39  40  41  41  42  42  43  43  44  44  45  45  46  46  46  46  46
 [55]  47  47  47  47  50  50  50  50  50  50  50  50  50  50  50  50  50  50
 [73]  50  51  51  51  51  51  51  51  51  51  52  52  52  53  53  53  53  53
 [91]  54  54  54  54  54  54  55  55  55  55  55  55  55  56  56  56  56  56
[109]  56  57  57  57  57  57  57  57  57  57  58  58  58  58  58  58  60  60
[127]  60  60  60  60  60  60  60  60  60  60  60  60  60  60  60  60  60  60
[145]  60  60  60  61  61  61  61  61  62  62  62  62  62  62  62  62  63  63
[163]  63  64  64  64  64  64  64  64  65  65  65  65  65  65  65  65  65  65
[181]  66  66  66  66  66  66  67  67  67  67  68  68  68  70  70  70  70  70
[199]  70  70  70  71  71  71  71  71  71  71  71  71  72  72  72  72  72  73
[217]  73  73  73  74  74  74  74  74  75  75  75  76  76  76  76  76  76  76
[235]  76  77  77  77  77  77  78  78  78  78  78  80  80  80  80  80  80  80
[253]  81  81  81  82  82  82  83  84  84  84  84  85  86  86  86  87  87  87
[271]  87  88  88  88  88  90  90  90  91  91  93  94  94  94  95  95  95  96
[289]  97  99 998 998 998 998 998 998 998
>
> # Fix it
> Math$ucalc[Math$ucalc==998] = NA
> # Fix dummy variables of frstlang at the data frame level too.
> Math$frstlang = factor(Math$frstlang)
> Math = na.omit(Math)
> dim(Math)
[1] 290   9
> attach(Math)
The following object is masked _by_ .GlobalEnv:

    frstlang

The following objects are masked from Math (pos = 3):

    calc, course, frstlang, hscalc, hsengl, hsgpa, precalc, sex,
    ucalc

>
> # Fit the model again, still calling it fullmodel1
> fullmodel1  = lm(ucalc ~ course + precalc + calc + hsgpa + hscalc + hsengl +
frstlang + sex)
Error in model.frame.default(formula = ucalc ~ course + precalc + calc +  :
  variable lengths differ (found for 'frstlang')
>
> length(frstlang)
[1] 297
> rm(frstlang)
> length(frstlang)
[1] 290
```

```
>
> fullmodel1  = lm(ucalc ~ course + precalc + calc + hsgpa + hscalc + hsengl +
frstlang + sex)
> summary(fullmodel1)

Call:
lm(formula = ucalc ~ course + precalc + calc + hsgpa + hscalc +
    hsengl + frstlang + sex)

Residuals:
    Min      1Q  Median      3Q     Max
-51.801  -6.911   1.825   8.845  31.892

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)    -26.62766   10.25376  -2.597 0.009905 **
courseElite     -8.92387    5.40848  -1.650 0.100069
courseMainstrm  -5.78575    4.61603  -1.253 0.211104
precalc          2.03123    0.59845   3.394 0.000788 ***
calc             0.77213    0.40292   1.916 0.056337 .
hsgpa            0.31636    0.09886   3.200 0.001532 **
hscalc           0.58927    0.09270   6.357 8.33e-10 ***
hsengl           0.10509    0.11304   0.930 0.353356
frstlangOther    5.81009    2.24539   2.588 0.010170 *
sexM            -1.35617    1.82179  -0.744 0.457250
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 14.43 on 280 degrees of freedom
Multiple R-squared:  0.3996,  Adjusted R-squared:  0.3803
F-statistic: 20.71 on 9 and 280 DF,  p-value: < 2.2e-16

>
> # That is a relief! At this point it is messy enough to
> # justify a clean re-start. Quit R.
```

```
> # Read and fix the data
> rm(list=ls())
> math =
read.table("http://www.utstat.utoronto.ca/~brunner/data/legal/math1.data.txt")
> # Clean the data
> math$hscalc[math$hscalc==999] = NA
> math$hscalc[math$hscalc==0] = NA
> math$hsengl[math$hsengl==0] = NA
> math$ucalc[math$ucalc==999] = NA
> math$ucalc[math$ucalc==998] = NA
> math$frstlang[math$frstlang=='French'] = 'Other'
> math$frstlang = factor(math$frstlang)
> math = na.omit(math)
> attach(math)
>
> fullmodel1  = lm(ucalc ~ course + precalc + calc + hsgpa + hscalc + hsengl +
frstlang + sex)
> summary(fullmodel1)

Call:
lm(formula = ucalc ~ course + precalc + calc + hsgpa + hscalc +
    hsengl + frstlang + sex)

Residuals:
    Min      1Q  Median      3Q     Max
-51.801  -6.911   1.825   8.845  31.892

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)   -26.62766   10.25376  -2.597 0.009905 **
courseElite    -8.92387    5.40848  -1.650 0.100069
courseMainstrm -5.78575    4.61603  -1.253 0.211104
precalc         2.03123    0.59845   3.394 0.000788 ***
calc            0.77213    0.40292   1.916 0.056337 .
hsgpa           0.31636    0.09886   3.200 0.001532 **
hscalc          0.58927    0.09270   6.357 8.33e-10 ***
hsengl          0.10509    0.11304   0.930 0.353356
frstlangOther   5.81009    2.24539   2.588 0.010170 *
sexM           -1.35617    1.82179  -0.744 0.457250
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.43 on 280 degrees of freedom
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F-statistic: 20.71 on 9 and 280 DF,  p-value: < 2.2e-16
```
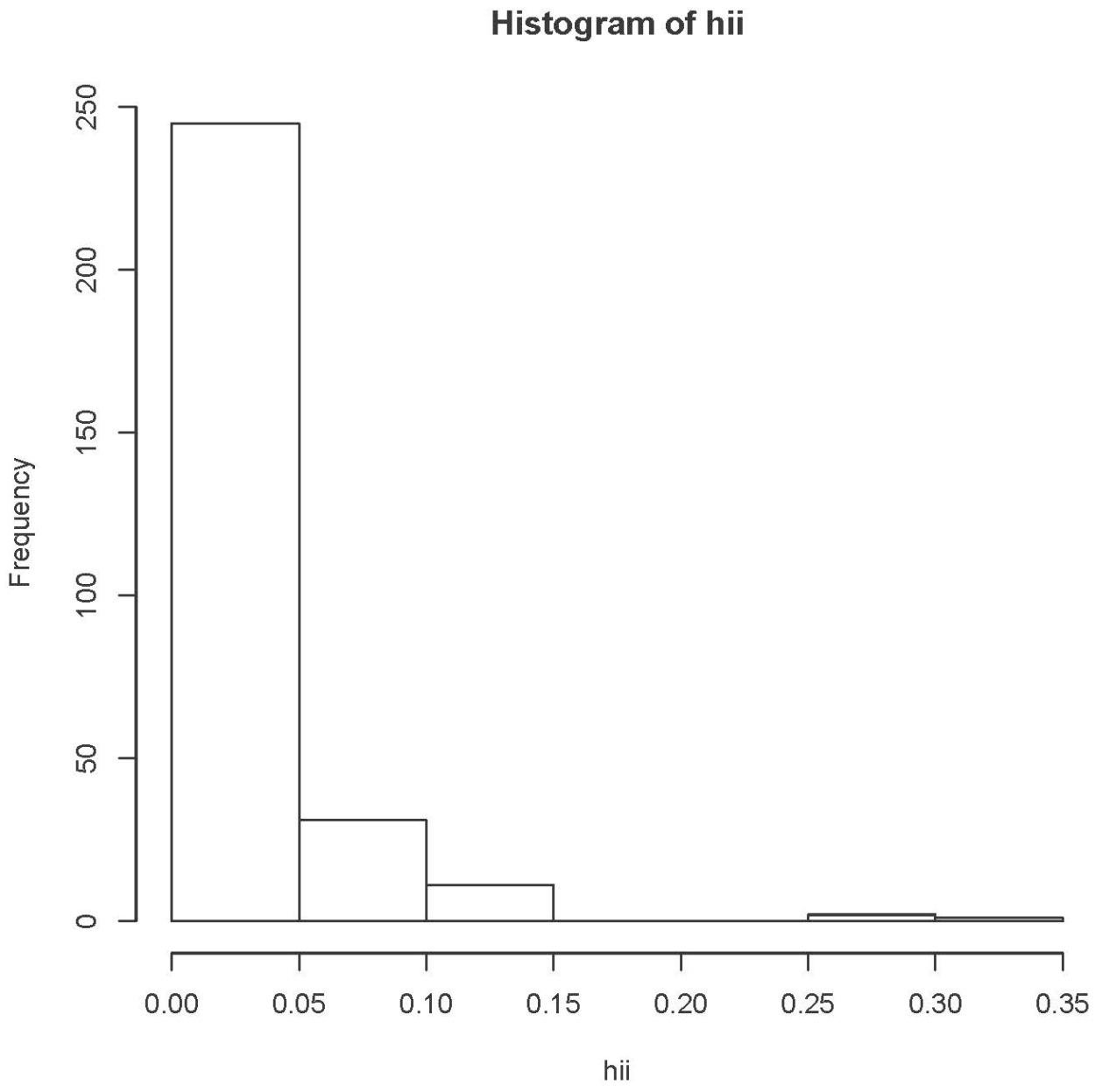
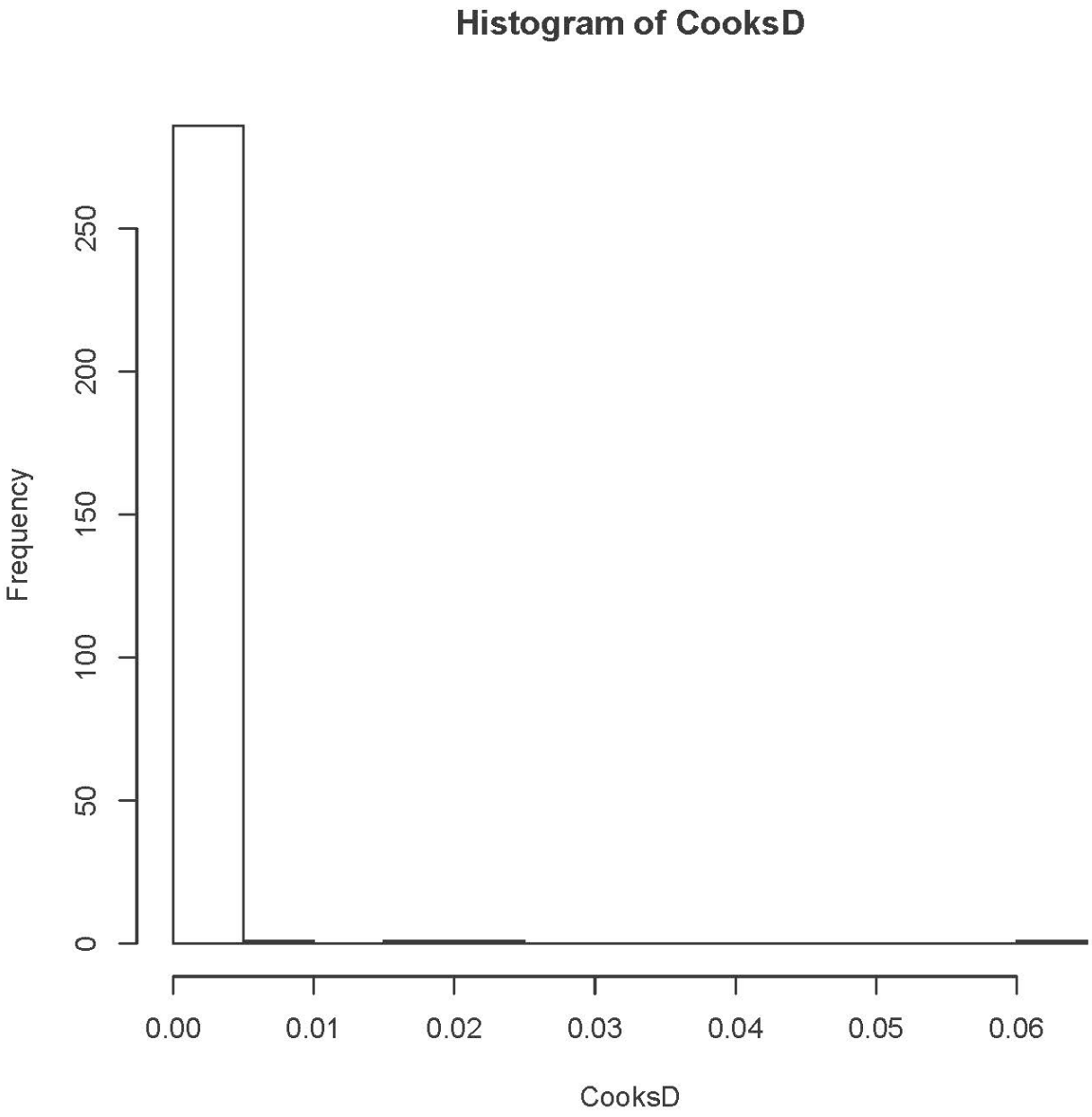

```
>
> ################ Influence Measures ###########
>
> # Leverage
> hii = hatvalues(fullmodel1)
> summary(hii); hist(hii)
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
0.008746 0.017470 0.025360 0.034480 0.039550 0.319500
```




```
> hii[hii>0.2]
      116       127       133
0.2837035 0.2743562 0.3195286
>
```

```
> hii[hii>0.2]
      116       127       133
0.2837035 0.2743562 0.3195286
>
> # Cook's Distance
> n = length(frstlang)
> k = length(coefficients(fullmodel1))
> ei = residuals(fullmodel1)
> s = summary(fullmodel1)$sigma
> CooksD = ei^2/((k+1)*s^2) * (hii/(1-hii))^2
> summary(CooksD); hist(CooksD)
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
0.000e+00 3.810e-06 2.028e-05 4.847e-04 9.199e-05 6.067e-02
```



```
> 4/n # Rule of thumb says watch out if Cook's D > 4/n
[1] 0.0137931
> CooksD[CooksD>0.01] # Based on histogram
       116        127        133
0.06067450 0.02046928 0.01675851
> hii[hii>0.2] # Again, for comparison
      116       127       133
0.2837035 0.2743562 0.3195286

>
> math[c(116,127,133),] # These rows, all the columns
      course precalc calc hsgpa hscalc hsengl ucalc frstlang sex
235 Mainstrm       4    2     0     60     82    51  English   F
256 Mainstrm       3    3     0     65     74    46    Other   F
263 Mainstrm       7    7     0     91     62    65    Other   M
> sort(hsgpa)
  [1]  0.0  0.0  0.0 66.0 66.0 69.0 69.8 70.0 70.5 70.7 70.7 71.8 72.2 72.3 72.3
 [16] 72.3 72.5 72.5 72.5 72.5 72.8 73.0 73.0 73.2 73.3 73.3 73.5 73.5 73.5 73.7
 [31] 73.7 73.8 73.8 74.0 74.0 74.2 74.3 74.3 74.5 74.5 74.5 74.5 74.5 74.7 74.7
 [46] 74.8 74.8 74.8 75.0 75.0 75.2 75.2 75.2 75.2 75.3 75.5 75.7 75.7 75.8 75.8
 [61] 75.8 76.0 76.0 76.2 76.2 76.2 76.2 76.3 76.3 76.3 76.5 76.7 76.7 76.7 76.8
 [76] 76.8 76.8 77.0 77.0 77.0 77.2 77.2 77.2 77.3 77.3 77.3 77.5 77.5 77.5 77.5
 [91] 77.5 77.5 77.7 77.7 77.8 78.0 78.0 78.0 78.0 78.0 78.0 78.2 78.2 78.2 78.2
[106] 78.2 78.3 78.3 78.3 78.3 78.3 78.3 78.3 78.3 78.3 78.5 78.5 78.5 78.5 78.7 78.7
[121] 78.7 78.7 78.8 78.8 79.0 79.0 79.3 79.3 79.3 79.3 79.5 79.5 79.5 79.5 79.7
[136] 79.7 79.8 79.8 80.0 80.0 80.0 80.0 80.0 80.0 80.0 80.2 80.2 80.2 80.3 80.3
[151] 80.5 80.5 80.5 80.7 80.7 80.7 80.7 80.8 80.8 80.8 81.0 81.0 81.0 81.0 81.0
[166] 81.0 81.0 81.0 81.2 81.2 81.2 81.2 81.3 81.3 81.5 81.7 81.7 81.7 81.7 81.8
[181] 81.8 81.8 81.8 82.0 82.0 82.0 82.0 82.0 82.2 82.2 82.2 82.2 82.3 82.3 82.3
[196] 82.5 82.5 82.7 82.7 82.8 82.8 83.0 83.0 83.0 83.2 83.3 83.3 83.3 83.5 83.7
[211] 83.8 83.8 84.0 84.0 84.0 84.2 84.3 84.5 84.7 84.7 84.7 84.7 84.8 85.2 85.2
[226] 85.3 85.8 85.8 86.2 86.3 86.5 86.5 86.7 86.7 86.7 86.7 86.8 86.8 86.8 86.8
[241] 87.0 87.3 87.3 87.3 87.8 88.0 88.2 88.3 88.3 88.3 88.5 88.5 88.5 88.7 88.7
[256] 88.7 89.0 89.2 90.2 90.3 90.3 90.5 90.5 90.8 91.0 91.0 91.0 91.5 91.5 91.5
[271] 91.5 91.8 92.3 92.3 92.3 92.3 92.5 92.5 92.7 93.2 93.3 93.5 93.5 93.7 93.7
[286] 94.0 94.3 95.0 95.8 96.2
>
> # Do the residuals reveal this?
> stdelres = rstudent(fullmodel1)
> n = length(ucalc)
> a = 0.05/n
> critval = qt(1-a/2,fullmodel1$df-1) # df = n-k-2
> critval # 3.807824
[1] 3.807824
> stdelres[abs(stdelres)>critval]
       86
-3.819437
> math[86,]
    course precalc calc hsgpa hscalc hsengl ucalc frstlang sex
176  Elite       3    6  69.8     76     60     1    Other   M
```

```
> # Fix those three zeros (just locally) and re-fit the model
> hsgpa[hsgpa==0] = NA

> fullmodel2 = lm(ucalc ~ course + precalc + calc + hsgpa + hscalc + hsengl +
frstlang + sex)
> summary(fullmodel2)

Call:
lm(formula = ucalc ~ course + precalc + calc + hsgpa + hscalc +
    hsengl + frstlang + sex)

Residuals:
    Min      1Q  Median      3Q     Max
-45.637  -6.557   1.183   8.517  30.945

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    -66.7948    11.5843  -5.766 2.16e-08 ***
courseElite     -8.3597     5.0888  -1.643  0.10156
courseMainstrm  -5.6157     4.3396  -1.294  0.19672
precalc          1.7374     0.5657   3.071  0.00235 **
calc             0.7117     0.3789   1.878  0.06140 .
hsgpa            1.5717     0.2211   7.110 9.89e-12 ***
hscalc           0.2513     0.1033   2.433  0.01560 *
hsengl          -0.3273     0.1264  -2.590  0.01011 *
frstlangOther    4.4895     2.1334   2.104  0.03625 *
sexM            -1.6280     1.7155  -0.949  0.34344
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.56 on 277 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.4737,  Adjusted R-squared:  0.4566
F-statistic:  27.7 on 9 and 277 DF,  p-value: < 2.2e-16

>
> # We are not done, but this illustrates the role of influence diagnostics
> # in data cleaning.
```
