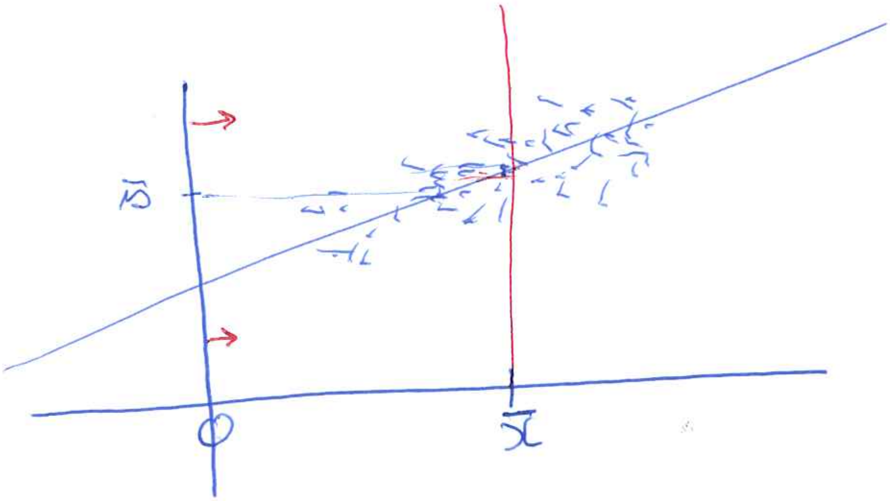
Centered model



Ex 1

$$
\begin{aligned}
y_i &= \beta_0 + \beta_1 x_i + \varepsilon_i \\
&= \beta_0 + \beta_1 (x_i - \bar{x} + \bar{x}) + \varepsilon_i \\
&= (\beta_0 + \beta_1 \bar{x}) + \beta_1 (x_i - \bar{x}) + \varepsilon_i \\
&= \beta_0^* + \beta_1^* (x_i - \bar{x}) + \varepsilon_i
\end{aligned}
$$

Ex 2

$$
\begin{aligned}
y_i &= \beta_0 + \beta_1 x_i + \beta_2 d_i + \varepsilon_i \\
&= \beta_0 + \beta_1 (x_i - \bar{x} + \bar{x}) + \beta_2 d_i + \varepsilon_i \\
&= \underbrace{(\beta_0 + \beta_1 \bar{x})}_{\beta_0^*} + \beta_1 (x_i - \bar{x}) + \beta_2 d_i + \varepsilon_i
\end{aligned}
$$

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	d	$E(y) = \beta_0^* + \beta_1(x-\bar{x}) + \beta_2 d$
Ex	1	$\beta_0^* + \beta_2 + \beta_1(x-\bar{x})$
Pl	0	$\beta_0^* + \beta_1(x-\bar{x})$

Ex 3

$$y_i = \beta_0 + \beta_1(x_{i1} - \bar{x}_1 + \bar{x}_1) + \beta_2(x_{i2} - \bar{x}_2 + \bar{x}_2) + \cdots + \beta_k(x_{ik} - \bar{x}_k + \bar{x}_k) + \varepsilon_i$$

$$= (\beta_0 + \beta_1\bar{x}_1 + \cdots + \beta_k\bar{x}_k) + \beta_1(x_{i1} - \bar{x}_1) + \cdots + \beta_k(x_{ik} - \bar{x}_k) + \varepsilon_i$$

How about Estimation, testing, CI

Hope
- $\hat{y}$, e unchanged
- only tests & CIs for β_0 affected

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These are all 1-1 linear transformations of the X values, and corresponding 1-1 transformation of βs.

$$y = X\beta + \varepsilon = \underbrace{XA^{-1}}_{X^*}\underbrace{A\beta}_{\beta^*} + \varepsilon$$

A is $(k+1) \times (k+1)$

FYI

$$\underbrace{\begin{pmatrix} 1 & \bar{x} \\ 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}}_{\beta} = \underbrace{\begin{pmatrix} \beta_0 + \beta_1 \bar{x} \\ \beta_1 \end{pmatrix}}_{\beta^*}$$

Ex3

$$\begin{pmatrix} 1 & \bar{x}_1 & \bar{x}_2 & \cdots & \bar{x}_k \\ 0 & 1 & 0 & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & \cdots & & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} = \begin{pmatrix} \beta_0 + \beta_1 \bar{x}_1 + \cdots + \beta_k \bar{x}_k \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix}$$

$$\begin{aligned} y &= XA^{-1}A\beta + \varepsilon \\ &= X^* \beta^* + \varepsilon \end{aligned}$$

$$\begin{aligned} b^* &= (X^{*\prime} X^*)^{-1} X^{*\prime} y \\ &= \left((XA^{-1})' XA^{-1}\right)^{-1} (XA^{-1})' y \\ &= \left(A^{-1\prime} X'XA^{-1}\right)^{-1} A^{-1\prime} X' y \\ &= A^{-1-1} (X'X)^{-1} A^{-1\prime -1} A^{-1\prime} X' y \\ &= A (X'X)^{-1} \underbrace{A' A'^{-1}}_{I} X' y = A(X'X)^{-1} X' y \\ &= Ab \end{aligned}$$

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$$b^* = Ab$$

$$\hat{y}^* = X^* b^* = \underbrace{XA^{-1}A}_{F} b = \hat{y}$$

$$e^* = e \quad s^2 \text{ unchanged}$$

$$H_0 : C\beta = \gamma \iff C^* \beta^* = \gamma$$

$$\underbrace{CA^{-1}}_{C^*} A\beta = \gamma$$

$$F^* = \cdots F$$

b^* in conjunction with dummy variables

$\cancel{E(y)=}\ \hat{y} = b_0 + b_1(x_{i1}-\bar{x}_1) + b_2(x_{i2}-\bar{x}_2) + b_3 d$

d	
1	$b_0+b_3 \quad + b_1(x_{i1}-\bar{x}_1) + b_2(x_{i2}-\bar{x}_2)$
0	$b_0 \quad + b_1(x_{i1}-\bar{x}_1) + b_2(x_{i2}-\bar{x}_2)$

b_0+b_3 & b_0 are sometimes called "corrected means"

$\hat{y} = b_0 + b_1(x-\bar{x}) + b_2 d + b_3(x-\bar{x})d$

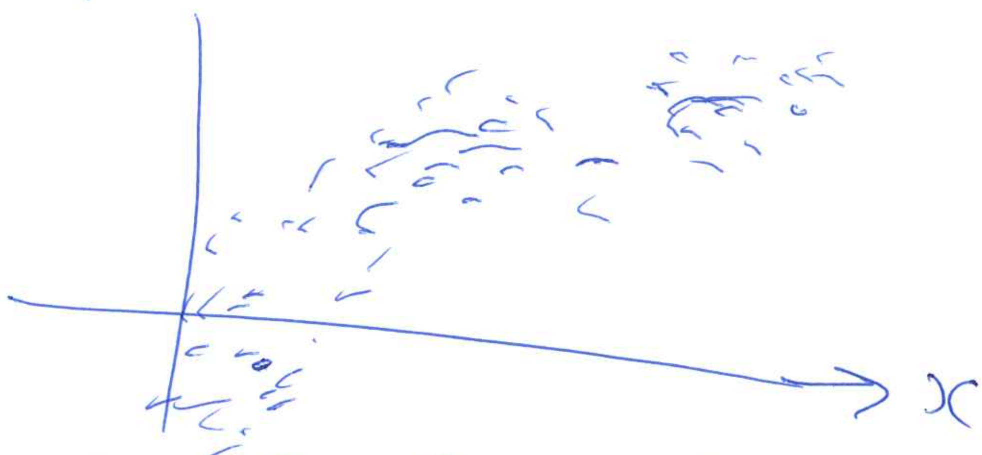
d	
1	$b_0+b_2 \quad + (b_1+b_3)(x-\bar{x})$
0	$b_0 \quad + b_1(x-\bar{x})$

Polynomial Regression

Taylor's Theorem says

$$g(x) = g(x_0) + g'(x_0)(x - x_0) + g''(x_0)\frac{(x-x_0)^2}{2!} + g'''(x_0)\frac{(x-x_0)^3}{3!} + \dots$$

If you see



Try $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i$

$$\frac{d}{dx} E(y) = \beta_1 + 2\beta_2 x$$

With x centered, β_1 is rate of change in $E(y)$ at average x