Name Jerry

Student Number

STA 302 Fall 2016 Quiz 2

3

1. (6 points) For simple linear regression (the model on the formula sheet, with $E(\epsilon_i) = 0$), derive the least squares estimates b_0 and b_1 that appear on the formula sheet; either formula for b_1 will do. There is no need to verify that an actual minimum is attained at these values; we will do that later. Show all your work.

$$\frac{\partial}{\partial \beta_0} \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2 = \sum_{i=1}^n \frac{\partial}{\partial \beta_0} (y_i - \beta_0 - \beta_1 x_i)^2$$

$$= \sum_{i=1}^n 2(y_i - \beta_0 - \beta_1 x_i)(-1) \stackrel{set}{=} 0$$

$$\Rightarrow \sum_{i=1}^n y_i - n\beta_0 - \beta_1 \sum_{i=1}^n x_i = 0$$

$$\Rightarrow \sum_{i=1}^n y_i = n\beta_0 + \beta_1 \sum_{i=1}^n x_i = n\beta_0 + n\beta_1 \bar{x}$$

$$\Rightarrow \bar{y} = \beta_0 + \beta_1 \bar{x} \Rightarrow \beta_0 = \bar{y} - \beta_1 \bar{x} \quad (1)$$

and

$$\frac{\partial}{\partial \beta_1} \sum_{i=1}^{} (y_i - \beta_0 - \beta_1 x_i)^2 = \sum_{i=1}^n 2(y_i - \beta_0 - \beta_1 x_i)(-x_i)$$

$$= -2 \sum_{i=1}^n (x_i y_i - \beta_0 x_i - \beta_1 x_i^2) \stackrel{set}{=} 0$$

$$\Rightarrow \sum_{i=1}^n x_i y_i - \beta_0 \sum_{i=1}^n x_i - \beta_1 \sum_{i=1}^n x_i^2 = 0$$

$$\Rightarrow \sum_{i=1}^n x_i y_i = \beta_0 n\bar{x} + \beta_1 \sum_{i=1}^n x_i^2 \quad \text{substituting } \beta_0 \text{ from (1)}$$

$$\Rightarrow \sum_{i=1}^n x_i y_i = (\bar{y} - \beta_1 \bar{x}) n\bar{x} + \beta_1 \sum_{i=1}^n x_i^2$$

$$= n\bar{x}\bar{y} - \beta_1 n\bar{x}^2 + \beta_1 \sum_{i=1}^n x_i^2$$

(over)

$$\Rightarrow \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} = \beta_1\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

Divide to solve for β_1, and call the solution

$$b_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$$ and $$b_0 = \bar{y} - b_1\bar{x}$$

Which appear on the formula sheet.

1 ~~2.~~ (2 points) Let the random variable Y have moment-generating function $M_Y(t)$. Prove the formula for $M_{aY}(t)$ given on the formula sheet.

$$M_{aY}(t) = E\left(e^{(aY)t}\right) = E\left(e^{Y(at)}\right) = M_Y(at)$$

2 ~~3.~~ (2 points) You did Exercise 1.9 in the text using R.

(a) Write b_0 in the space below. The answer is a number from your printout. On your printout, circle the number and write *Question 3a* beside it.

$b_0 = -1.60185$

(b) Write b_1 in the space below. The answer is a number from your printout. On your printout, circle the number and write *Question 3b* beside it.

$b_1 = 0.03261$

Please turn in your printout with your quiz paper. Make sure your name and student number are on the printout.