

Regression diagnostics with R*

```
> sat = read.table("http://www.utstat.utoronto.ca/~brunner/data/legal/openSAT.data.txt")
> head(sat)
  VERBAL MATH GPA
1    623  509 2.6
2    454  471 2.3
3    643  700 2.4
4    585  719 3.0
5    719  710 3.1
6    693  643 2.9
> mod1 = lm(GPA ~ VERBAL+MATH, data=sat); summary(mod1)
```

Call:

```
lm(formula = GPA ~ VERBAL + MATH, data = sat)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|----------|---------|---------|---------|
| -2.24875 | -0.35113 | 0.04659 | 0.38745 | 1.03527 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|-----------|------------|---------|--------------|
| (Intercept) | 0.6062975 | 0.4414062 | 1.374 | 0.171 |
| VERBAL | 0.0023072 | 0.0005522 | 4.178 | 4.42e-05 *** |
| MATH | 0.0009999 | 0.0006093 | 1.641 | 0.102 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5484 on 197 degrees of freedom

Multiple R-squared: 0.1161, Adjusted R-squared: 0.1071

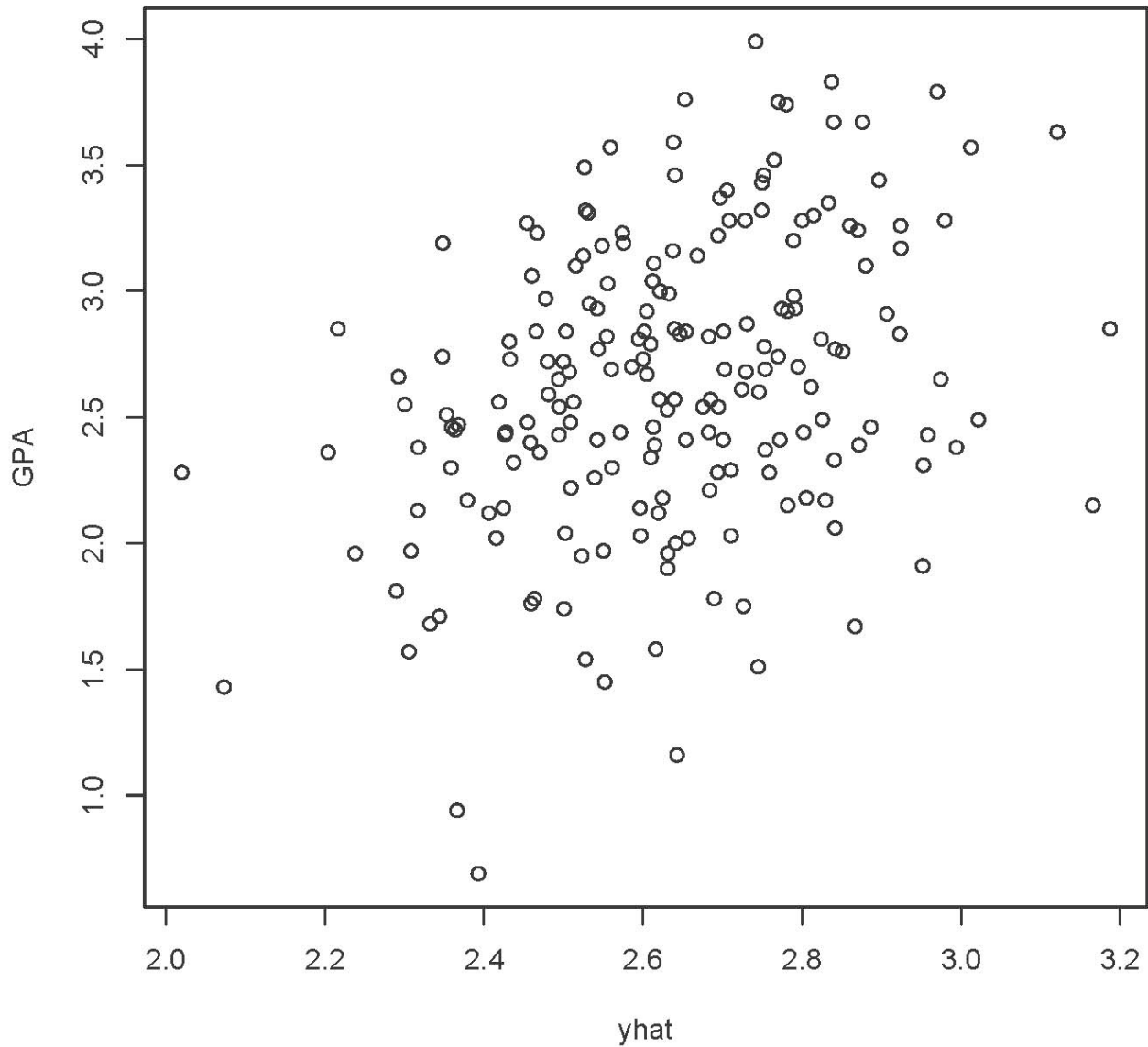
F-statistic: 12.93 on 2 and 197 DF, p-value: 5.284e-06

```
> attach(sat) # Make variable names accessible
```

```
>
```

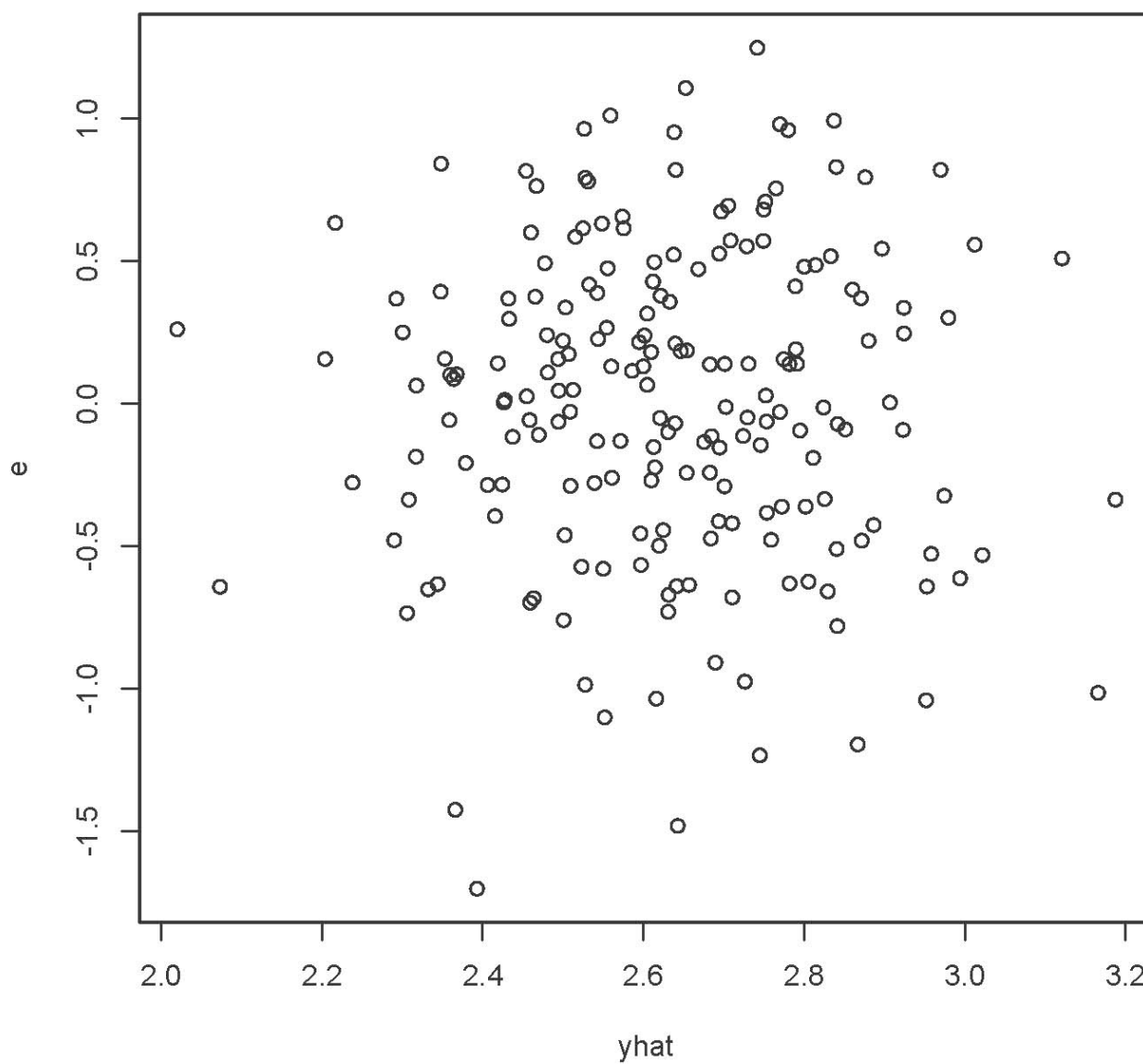
* Copyright information is on the last page.

```
> # Plot y-hat versus y
> yhat = mod1$fitted.values
> plot(yhat,GPA)
>
```



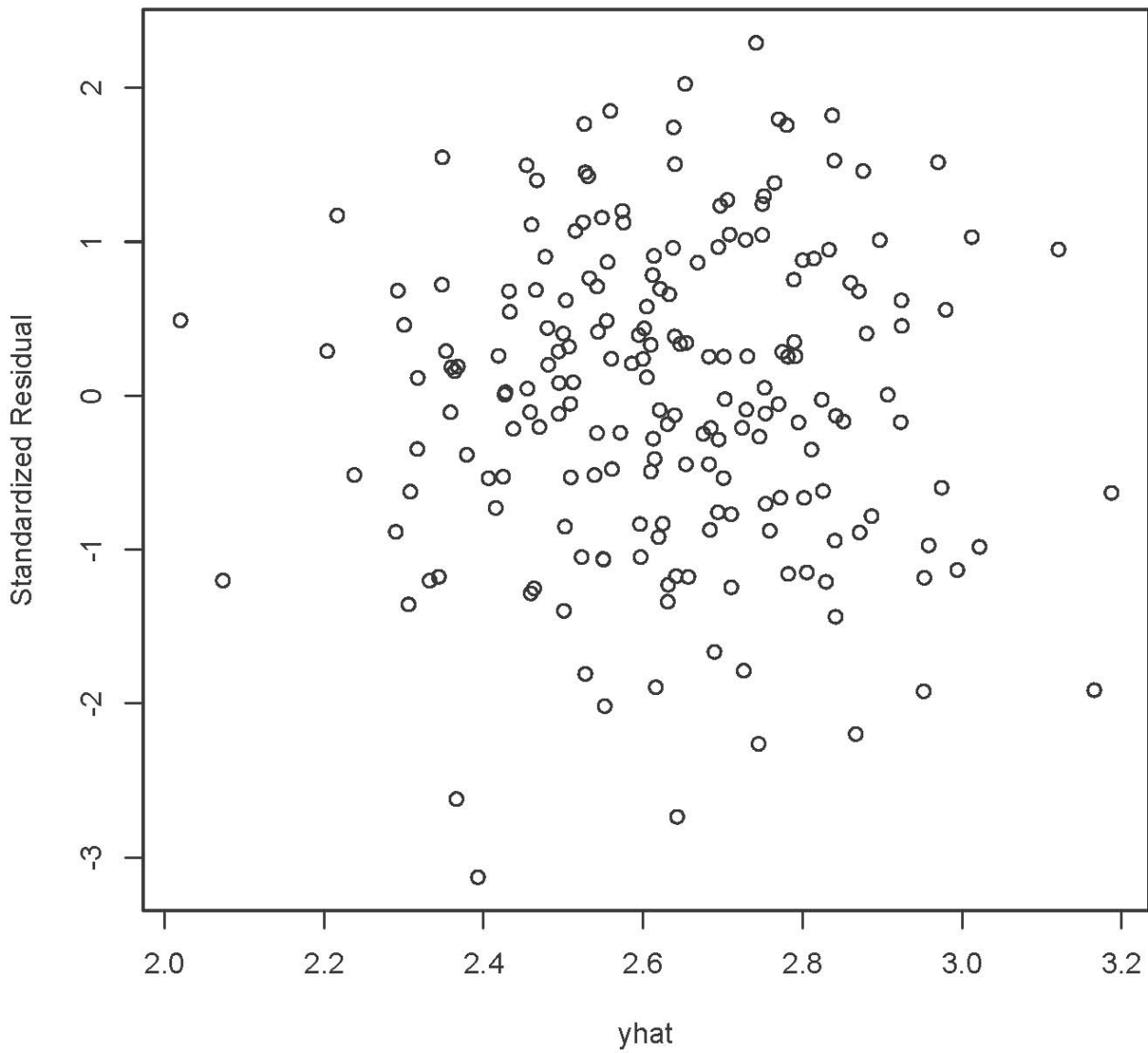
```
> cor(GPA,yhat)^2 # r-squared - R-squared
[1] 0.1160179
```

```
> # Plot y-hat versus residuals
> e = mod1$residuals
> plot(yhat,e)
```



```
> cor(yhat,e) # Zero
[1] 2.898153e-16
```

```
> # Compare plot of standardized residuals
> sr = rstandard(mod1)
> plot(yhat,sr,ylab='Standardized Residual')
```



```

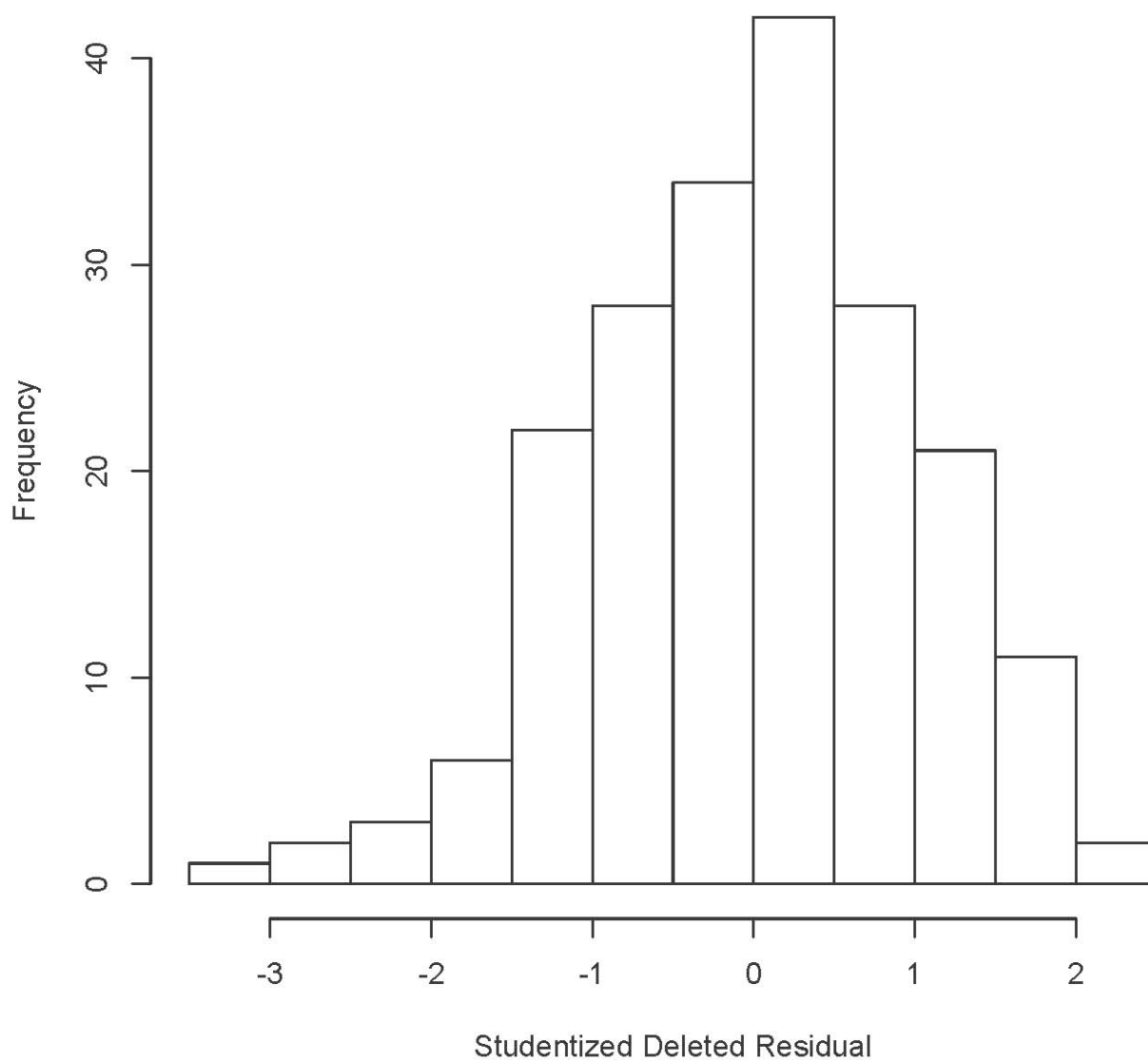
> # Three look like possible outliers: Investigate
> id = 1:200
> suspect = id[sr < -2.5]
> cbind(sat[suspect,],yhat[suspect],sr[suspect])
  VERBAL MATH  GPA yhat[suspect] sr[suspect]
20   497  640 0.69      2.392965   -3.128203
51   645  548 1.16      2.642644   -2.735853
73   516  569 0.94      2.365985   -2.620870

> # Studentized deleted residuals are t-statistics
> sdr = rstudent(mod1) # Studentized deleted residuals
> # Bonferroni critical value for n=200 tests, at joint alpha = 0.05 level
> dfe = mod1$df.residual; dfe
[1] 197
> alpha = 0.05; a = alpha/200; bcrit = qt(1-a/2,dfe-1); bcrit
[1] 3.730706
> summary(sdr)
   Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
-3.201000 -0.672800  0.048340 -0.001731  0.710900  2.317000

```

```
> hist(sdr,xlab='Studentized Deleted Residual',  
+ main='Histogram of Studentized Deleted Residuals')
```

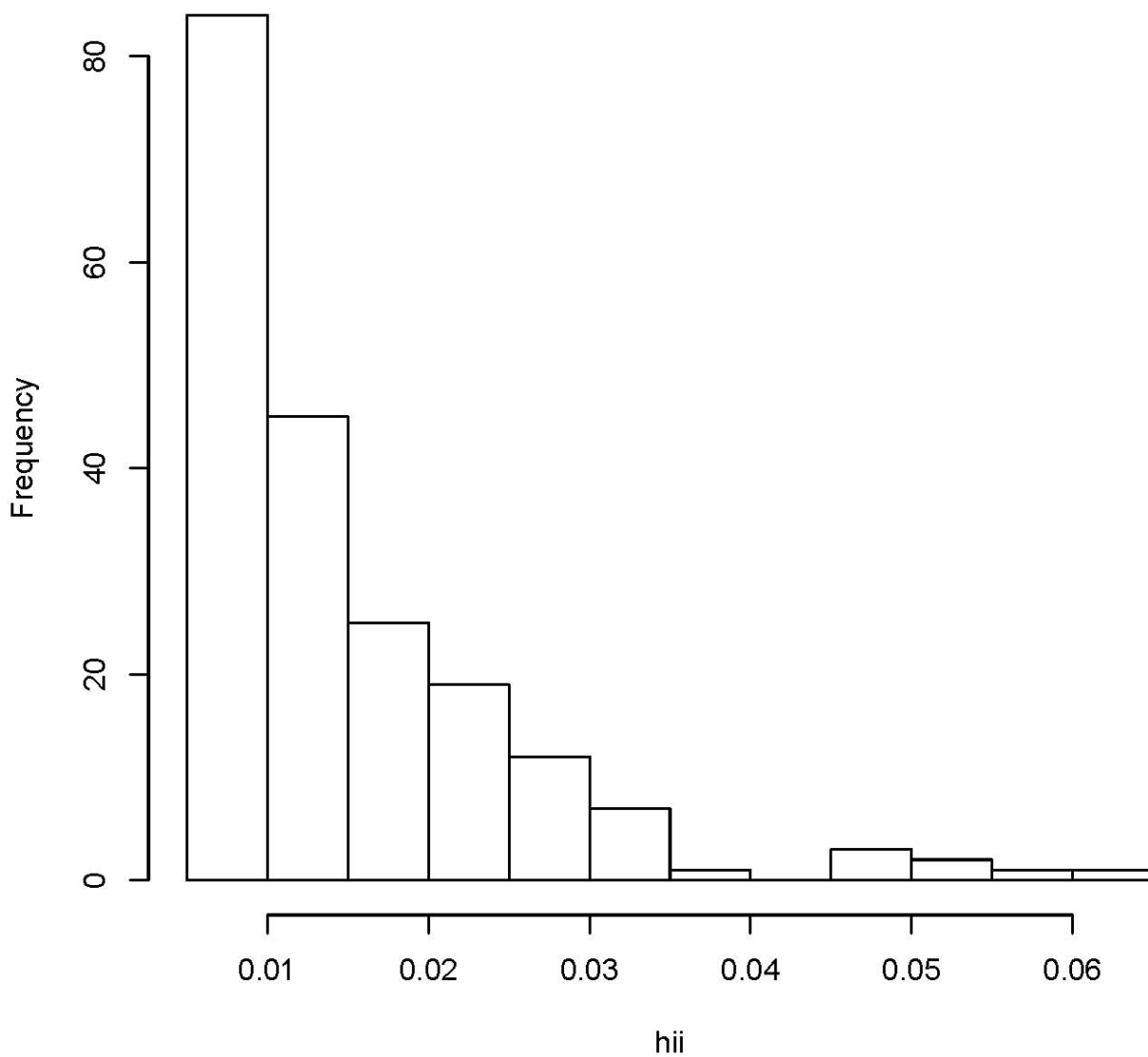
Histogram of Studentized Deleted Residuals



Should


```
> # Should we worry about normality? Not if maximum diagonal element
of the hat matrix is less than 0.2
> hii = hatvalues(mod1)
> summary(hii) # Max should be less than 0.2 for robustness.
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.005071 0.007824 0.011310 0.015000 0.019100 0.062370
> hist(hii)
```

Histogram of hii



Trees Data

```
> rm(list=ls()) # Remove everything to start
> head(trees)
  Girth Height Volume
1   8.3    70   10.3
2   8.6    65   10.3
3   8.8    63   10.2
4  10.5    72   16.4
5  10.7    81   18.8
6  10.8    83   19.7
> attach(trees)
> mod1 = lm(Volume ~ Girth + Height)
> summary(mod1)
```

Call:
lm(formula = Volume ~ Girth + Height)

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---------|---------|---------|--------|--------|
| -6.4065 | -2.6493 | -0.2876 | 2.2003 | 8.4847 |

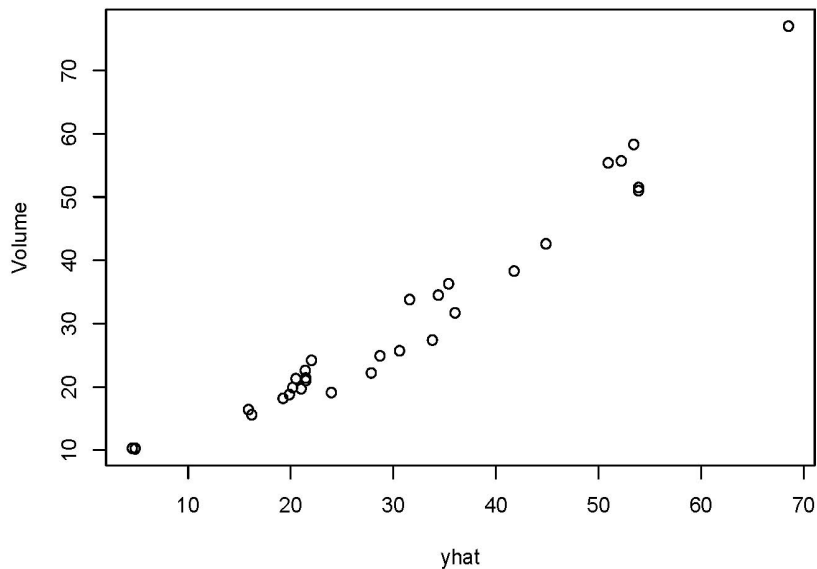
Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) | |
|-------------|----------|------------|---------|----------|-----|
| (Intercept) | -57.9877 | 8.6382 | -6.713 | 2.75e-07 | *** |
| Girth | 4.7082 | 0.2643 | 17.816 | < 2e-16 | *** |
| Height | 0.3393 | 0.1302 | 2.607 | 0.0145 | * |

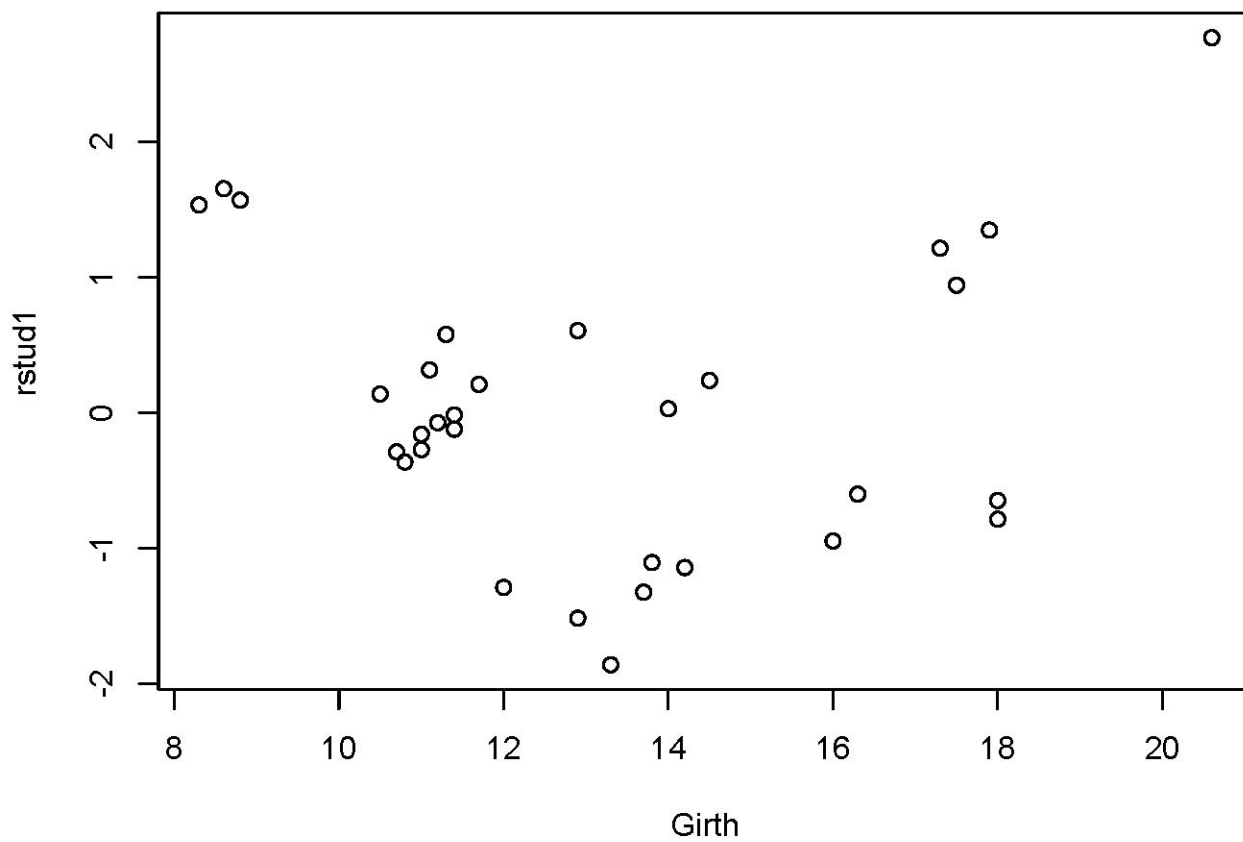
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.882 on 28 degrees of freedom
Multiple R-squared: 0.948, Adjusted R-squared: 0.9442
F-statistic: 255 on 2 and 28 DF, p-value: < 2.2e-16

```
>
> plot(yhat,Volume)
```



```
> rstud1 = rstudent(mod1) # Studentized deleted residuals
> plot(Girth,rstud1)
```



```

> plot(Height,rstud1) # No pattern
> Girthsq = Girth^2 # Polynomial term -- literally square it
> mod2 = lm(Volume ~ Girth + Girthsq + Height); summary(mod2)

Call:
lm(formula = Volume ~ Girth + Girthsq + Height)

Residuals:
    Min       1Q   Median       3Q      Max
-4.2928 -1.6693 -0.1018  1.7851  4.3489

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -9.92041    10.07911  -0.984  0.333729
Girth       -2.88508     1.30985  -2.203  0.036343 *
Girthsq      0.26862     0.04590   5.852  3.13e-06 ***
Height       0.37639     0.08823   4.266  0.000218 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.625 on 27 degrees of freedom
Multiple R-squared:  0.9771, Adjusted R-squared:  0.9745
F-statistic: 383.2 on 3 and 27 DF, p-value: < 2.2e-16
> a = (0.9771-0.948)/(1-0.948); a # Proportion of remaining variation
[1] 0.5596154

> 5.852^2/(27+5.852^2) # a = rF/(n-p+rF)
[1] 0.5591542

> rstud2 = rstudent(mod2)
> plot(Girth,rstud2) # Nothing
> plot(Height,rstud2) # Nothing
>
> # Look at prediction intervals
> cbind(Volume[1:5],predict(mod1,interval='predict')[1:5,])
      fit      lwr      upr
1 10.3  4.837660 -3.561809 13.23713
2 10.3  4.553852 -3.962908 13.07061
3 10.2  4.816981 -3.809144 13.44311
4 16.4 15.874115  7.690594 24.05764
5 18.8 19.869008 11.451358 28.28666
Warning message:
In predict.lm(mod1, interval = "predict") :
  predictions on current data refer to _future_ responses

>
> cbind(Volume[1:5],predict(mod2,interval='predict')[1:5,])
      fit      lwr      upr
1 10.3 10.985950  4.902269 17.06963
2 10.3  9.600406  3.566753 15.63406
3 10.2  9.205421  3.163767 15.24708
4 16.4 16.501775 10.954762 22.04879
5 18.8 20.451204 14.746331 26.15608
Warning message:
In predict.lm(mod2, interval = "predict") :
  predictions on current data refer to _future_ responses

```

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