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Student Number

STA 302f 2015 Quiz 7

Let $Y_1, \ldots, Y_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution.

1. (1 point) What is the distribution of $\overline{Y}$? You don't need to prove anything; just write the answer down.

$$\bar{Y} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

2. (1 point) What is the distribution of $\left(\frac{\overline{Y}-\mu}{\sigma/\sqrt{n}}\right)$? You don't need to prove anything; just write the answer down.

$$N(0,1)$$

3. (8 points) Show that

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1),$$

In homework, you proved that $\overline{Y}$ was independent of $S^2 = \frac{\sum_{i=1}^n (Y_i - \overline{Y})^2}{n-1}$, so you don't have do it again; just use the result. **For full marks, state clearly where you use the independence of $\overline{Y}$ and S^2.**

$$\sum_{i=1}^n (Y_i - \mu)^2 = \sum_{i=1}^n (Y_i - \bar{Y} + \bar{Y} - \mu)^2$$

$$= \sum_{i=1}^n (Y_i - \bar{Y})^2 + 2\sum_{i=1}^n (Y_i - \bar{Y})(\bar{Y} - \mu) + \sum_{i=1}^n (\bar{Y} - \mu)^2$$

$$= \sum_{i=1}^n (Y_i - \bar{Y})^2 + 2(\bar{Y} - \mu)\underbrace{\cancel{\sum_{i=1}^n (Y_i - \bar{Y})}}_{0} + n(\bar{Y} - \mu)^2$$

$$\text{So } \frac{1}{\sigma^2}\sum_{i=1}^n (Y_i - \mu)^2 = \frac{1}{\sigma^2}\sum_{i=1}^n (Y_i - \bar{Y})^2 + \frac{n(\bar{Y} - \mu)^2}{\sigma^2}$$

$$\Rightarrow \sum_{i=1}^n \left(\frac{Y_i - \mu}{\sigma}\right)^2 = \frac{(n-1)S^2}{\sigma^2} + \left(\frac{\bar{Y} - \mu}{\sigma/\sqrt{n}}\right)^2$$

$$W = W_1 + W_2$$

(over)

$$\sum_{i=1}^{n}\left(\frac{Y_i - \mu}{\sigma}\right)^2 = \frac{(n-1)S^2}{\sigma^2} + \left(\frac{\bar{Y} - \mu}{\sigma/\sqrt{n}}\right)^2$$

$$W \quad = \quad W_1 \quad + \quad W_2$$

$W \sim \chi^2(n)$ because it is the sum of n independent squared standard normals.

$W_2 \sim \chi^2(1)$ because it is the square of a standard normal

W_1 & W_2 are independent, because W_1 is a function of S^2 and W_2 is a function of $\bar{Y}$, and functions of independent random variables are independent. This is where I use independence.

So by the formula sheet W_1 has a chi-squared distribution with $df = n-1$

$$W_1 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

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