

Name Jerry

Student Number \_\_\_\_\_

## STA 302f 2015 Quiz 6

1. (5 points) Let the random vector  $\mathbf{y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ , and  $\mathbf{z} = \boldsymbol{\Sigma}^{-1/2}(\mathbf{y} - \boldsymbol{\mu})$ . Using moment-generating functions, derive the distribution of the random vector  $\mathbf{z}$ . Finish your answer with a clear statement of the distribution of  $\mathbf{z}$ .

$$\begin{aligned} M_{\mathbf{y}-\boldsymbol{\mu}}(\mathbf{t}) &= e^{-\mathbf{t}'\boldsymbol{\mu}} M_{\mathbf{y}}(\mathbf{t}) = e^{-\mathbf{t}'\boldsymbol{\mu}} e^{\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\boldsymbol{\Sigma}\mathbf{t}} \\ &= e^{\frac{1}{2}\mathbf{t}'\boldsymbol{\Sigma}\mathbf{t}}, \text{ and so} \end{aligned}$$

$$\begin{aligned} M_{\boldsymbol{\Sigma}^{-1/2}(\mathbf{y}-\boldsymbol{\mu})}(\mathbf{t}) &= M_{\mathbf{y}-\boldsymbol{\mu}}(\boldsymbol{\Sigma}^{-1/2}'\mathbf{t}) \\ &= e^{\frac{1}{2}(\boldsymbol{\Sigma}^{-1/2}'\mathbf{t})'\boldsymbol{\Sigma}(\boldsymbol{\Sigma}^{-1/2}'\mathbf{t})} \\ &= e^{\frac{1}{2}\mathbf{t}'\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{-1/2}'\mathbf{t}} = e^{\frac{1}{2}\mathbf{t}'\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{-1/2}'\mathbf{t}} \\ &= e^{\frac{1}{2}\mathbf{t}'(\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\Sigma}^{1/2})(\boldsymbol{\Sigma}^{1/2}\boldsymbol{\Sigma}^{-1/2})\mathbf{t}} \quad \text{OHAY to skip two steps} \\ &= e^{\frac{1}{2}\mathbf{t}'\mathbf{t}} \end{aligned}$$

MGF of  $N_p(0, I_p)$

2. (5 points) In homework, you used R to estimate the regression coefficients for the `statclass` data.

(a) Write  $\hat{\beta}_0$ ,  $\hat{\beta}_1$ ,  $\hat{\beta}_2$  and  $\hat{\beta}_3$  in the space below. Copy the numbers from your printout.

$$\hat{\beta}_0 = 9.14, \hat{\beta}_1 = 5.87, \hat{\beta}_2 = -2.93, \hat{\beta}_3 = \del{73.58} \\ 0.32$$

(b) What is the predicted final exam score for a student with a Quiz Average of 8, a Computer Average of 4, and a Midterm score of 90? Show a little work.

$$9.14 + (5.87)(8) + (-2.93)(4) + (0.32)(90) \\ = 73.58$$

(c) Circle  $\hat{\beta}_2$  on your printout, and write " $\hat{\beta}_2$ " beside it. Attach the *complete* printout to your quiz. Make sure your name and student number are written clearly on the printout.