Name Jenny

Student Number

STA 302f 2015 Quiz 5

1. (5 points) For the general linear regression model on the formula sheet, suppose that $(\mathbf{X}'\mathbf{X})^{-1}$ exists. Show either that

    (a) The columns of $\mathbf{X}$ are linearly independent, or

    (b) The columns of $\mathbf{X}$ are linearly dependent.

    Pick one and prove it.

Columns of X linearly independent means $Xa = 0$ implies $a = 0$

$$Xa = 0 \Rightarrow X'Xa = X'0 = 0$$

$$\Rightarrow \underbrace{(X'X)^{-1}X'X}_{I}a = (X'X)^{-1}0 = 0$$

$$\Rightarrow a = 0 \quad \blacksquare$$

2. (5 points) For simple linear regression through the origin, the model is $Y_i = \beta_1 x_i + \epsilon_i$ for $i = 1, \dots, n$, where $\epsilon_1, \dots, \epsilon_n$ are a random sample from a distribution with expected value zero and variance σ^2. The numbers $x_1, \dots, x_n$ are known, observed constants, while the parameters β_1 and σ^2 are unknown constants.

(a) Viewing this model as a special case of the general linear regression model, what is the **X** matrix?

$$\underset{\sim}{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

(b) Clearly, $\widehat{\beta}_1 = \widehat{\boldsymbol{\beta}}$ for this problem. Calculate $\widehat{\boldsymbol{\beta}}$, expressing the answer in terms of x_i and y_i values, and the sample size n. **Circle your final answer.** Show your work. Answers that could not be computed from a set of x_i and y_i values will get no marks.

$$X'X = \sum_{i=1}^{n} x_i^2, \quad (X'X)^{-1} = \frac{1}{\sum x_i^2}$$

$$X'Y = \sum_{i=1}^{n} x_i y_i$$

$$\hat{\beta} = (X'X)^{-1} X'Y = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$$