Name Jerry

Student Number ____________

STA 302f 2015 Quiz 4

1. (5 points) Let $\mathbf{Y}$ be a $p \times 1$ random vector with $E(\mathbf{Y}) = \boldsymbol{\mu}_y$ and $cov(\mathbf{Y}) = \boldsymbol{\Sigma}_y$. Let $\mathbf{T}$ be a $q \times 1$ random vector with $E(\mathbf{T}) = \boldsymbol{\mu}_t$ and $cov(\mathbf{T}) = \boldsymbol{\Sigma}_t$. Note that in general, $p \neq q$.

   Only one of the following statements is always true; the others are not true in general. Choose the true statement and prove it, starting with the definition of $cov(\mathbf{Y}, \mathbf{T})$ on the formula sheet.

   (a) $cov(\mathbf{Y}, \mathbf{T}) = cov(\mathbf{T}, \mathbf{Y})$.
   (b) $cov(\mathbf{Y}, \mathbf{T}) = \mathbf{0}$.
   (c) $cov(\mathbf{Y}, \mathbf{T}) = E(\mathbf{Y}\mathbf{T}') - E(\mathbf{Y})E(\mathbf{T})'$.
   (d) $cov(\mathbf{Y}, \mathbf{T}) = \boldsymbol{\Sigma}_y \boldsymbol{\Sigma}_t$.

(c) $Cov(Y, T) = E\{(Y - \mu_y)(T - \mu_t)'\}$

$= E\{YT' - Y\mu_t' - \mu_y T' + \mu_y \mu_t'\}$

$= E(YT') - E(Y)\mu_t' - \mu_y E(T') - \mu_y \mu_t'$

$= E(YT') - \mu_y \mu_t' - \mu_y \mu_t' + \mu_y \mu_t'$

$= E(YT') - \mu_y \mu_t'$

No marks for a, b or d

2. (5 points) The simple linear regression model is $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ for $i = 1, \ldots, n$, where $\epsilon_1, \ldots, \epsilon_n$ are a random sample from a distribution with expected value zero and variance σ^2. The numbers $x_1, \ldots, x_n$ are known, observed constants, while the parameters β_0 β_1 and σ^2 are unknown constants (parameters).

(a) Find $E(Y_i)$. Show a little work.

$$E(Y_i) = E(\beta_0 + \beta_1 x_i + \varepsilon_i) = \beta_0 + \beta_1 x_i + E(\varepsilon_i)$$
$$= \beta_0 + \beta_1 x_i$$

(b) Find $E(\overline{Y})$. Show your work.

$$E(\bar{Y}) = E\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(Y_i)$$
$$= \frac{1}{n}\sum_{i=1}^{n}(\beta_0 + \beta_1 x_i) = \beta_0 + \beta_1 \bar{x}$$

(c) In homework, you obtained the least-squares estimate $\widehat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \overline{x})(Y_i - \overline{Y})}{\sum_{i=1}^n (x_i - \overline{x})^2}$. Is $\widehat{\beta}_1$ an unbiased estimate of β_1? Answer Yes or No and show your work.

$$E(\hat{\beta}_1) = E\left\{\frac{\sum_{i=1}^{n}(x_i - \bar{x})(Y_i - \bar{Y})}{\sum (x_i - \bar{x})^2}\right\}$$

$$= \frac{\sum_{i=1}^{n}(x_i - \bar{x})E(Y_i - \bar{Y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})\left(E(Y_i) - E(\bar{Y})\right)}{\sum (x_i - \bar{x})^2}$$

$$= \frac{\sum_{i=1}^{n}(x_i - \bar{x})\left(\cancel{\beta_0} + \beta_1 x_i - (\cancel{\beta_0} + \beta_1 \bar{x})\right)}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$= \frac{\sum_{i=1}^{n}(x_i - \bar{x})\beta_1(x_i - \bar{x})}{\sum (x_i - \bar{x})^2} = \beta_1 \frac{\cancel{\sum (x_i - \bar{x})^2}}{\cancel{\sum (x_i - \bar{x})^2}} = \beta_1$$

Unbiased