Name Jenny

Student Number

STA 302f 2015 Quiz 2

1. (4 points) Let $Y = X_1 + X_2$, where X_1 and X_2 are independent, $X_1 \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. Use moment-generating functions to find the distribution of X_2.

$$M_Y(t) \overset{\text{Independ.}}{=} M_{X_1}(t) \quad M_{X_2}(t)$$

$$\Rightarrow e^{(\mu_1+\mu_2)t + \frac{1}{2}(\sigma_1^2+\sigma_2^2)t^2} = e^{\mu_1 t + \frac{1}{2}\sigma_1^2 t^2} \cdot M_{X_2}(t)$$

$$\Rightarrow \cancel{e^{\mu_1 t + \frac{1}{2}\sigma_1^2 t^2}} \times e^{\mu_2 t + \frac{1}{2}\sigma_2^2 t^2} = \cancel{e^{\mu_1 t + \frac{1}{2}\sigma_1^2 t^2}} \cdot M_{X_2}(t)$$

$$\Rightarrow M_{X_2}(t) = e^{\mu_2 t + \frac{1}{2}\sigma_2^2 t^2}$$

MGF of $N(\mu_2, \sigma_2^2)$

2. (3 points) Prove that if a set of vectors includes $\mathbf{0}$, the set is linearly dependent.

Denote the set of vectors by $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_p$, and suppose $\underset{\sim}{x}_j = \underset{\sim}{0}$. Let $a_j = 1$ and all the other $a_i = 0$. Then

$$a_1 \underset{\sim}{x}_1 + \cdots + a_j \underset{\sim}{x}_j + \cdots + a_p \underset{\sim}{x}_p = \underset{\sim}{0}$$ with

not all $a_i = 0$, satisfying the definition of linear dependence.

There will be a formula sheet.

3. (3 points) Let $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$. Give a non-zero vector $\mathbf{x}$ with $\mathbf{A}\mathbf{x} = \mathbf{0}$. Carry out the matrix multiplication to show that your $\mathbf{x}$ works.

Let $\underset{\sim}{x} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$