General Linear Test with R*

```
> options(scipen=999) # To avoid scientific notation
> kars =
read.table("http://www.utstat.utoronto.ca/~brunner/data/legal/mcars4.data")
> head(kars)

  Cntry lper100k weight length
1    US     19.8   2178   5.92
2 Japan      9.9   1026   4.32
3    US     10.8   1188   4.27
4    US     12.5   1444   5.11
5    US     12.5   1485   5.03
6    US     12.5   1485   5.03

> attach(kars) # Variables are now available by name
>
> fullmodel = lm(lper100k ~ weight+length+Cntry)
```

Origin	c1	c2	$E(Y\|\mathbf{X}=\mathbf{x}) = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3C_1 + \beta_4C_2$
Europe	0	0	$\beta_0 + \beta_1X_1 + \beta_2X_2$
Japan	1	0	$(\beta_0 + \beta_3) + \beta_1X_1 + \beta_2X_2$
U.S.	0	1	$(\beta_0 + \beta_4) + \beta_1X_1 + \beta_2X_2$

```
> summary(fullmodel)

Call:
lm(formula = lper100k ~ weight + length + Cntry)

Residuals:
    Min      1Q  Median      3Q     Max
-4.5063 -0.8813  0.0147  1.3043  2.9432

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.789215   2.855736  -2.027 0.045441 *
weight       0.005457   0.001472   3.707 0.000352 ***
length       2.345968   0.980329   2.393 0.018676 *
CntryJapan   0.506517   0.660158   0.767 0.444826
CntryUS     -1.487722   0.575633  -2.584 0.011274 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.703 on 95 degrees of freedom
Multiple R-squared:  0.7431,    Adjusted R-squared:  0.7323
F-statistic: 68.71 on 4 and 95 DF,  p-value: < 0.00000000000000022
```

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```
> # Test Country controlling for length and weight.
```

H_0: $\beta_3 = \beta_4 = 0$

$$F = \frac{(\mathbf{C}\widehat{\boldsymbol{\beta}} - \mathbf{t})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\boldsymbol{\beta}} - \mathbf{t})}{q\,MSE}$$

```
> LL = rbind(c(0,0,0,1,0),
+            c(0,0,0,0,1))
> V = vcov(fullmodel) # Don't need MSE because it's already in V
> q=dim(LL)[1]
> betahat = fullmodel$coefficients
> Cbeta = LL %*% betahat; center = solve(LL %*% V %*% t(LL))
> Fstat = as.numeric( t(Cbeta) %*% center %*% Cbeta ) / q ; Fstat
[1] 6.89995
> dfe = fullmodel$df.residual; dfe
[1] 95
> pval = 1-pf(Fstat,q,dfe); pval
[1] 0.001592274

> # A handy function for the general linear test: Use it freely
> source("http://www.utstat.toronto.edu/~brunner/Rfunctions/ftest.txt")
> ftest # To see function definition

function(model,L,h=0)
# General linear test of H0: L beta = h
    {
    BetaHat = model$coefficients
    dimL = dim(L)
    if(length(BetaHat) != dimL[2]) stop("Sizes of L and Beta are incompatible")
    r = dimL[1]
    if(qr(L)$rank != r) stop("Rows of L must be linearly independent.")
    out = numeric(4)
    names(out) = c("F","df1","df2","p-value")
    dfe = df.residual(model)
    diff = L%*%BetaHat-h
    fstat = t(diff) %*% solve(L%*%vcov(model)%*%t(L)) %*% diff / r
    # Note vcov = MSE * XtXinv
    fstat = as.numeric(fstat)
    out[1] = fstat; out[2]=r; out[3]=dfe
    out[4] = 1-pf(fstat,r,dfe)
    return(out)
    }
```

```
> Fstat # Again
[1] 6.89995

>
> ftest(fullmodel,LL)
           F          df1          df2      p-value
 6.899949667  2.000000000 95.000000000  0.001592274
>
> # Test weight and length controlling for country
>
> L2 = rbind(c(0,1,0,0,0),
+            c(0,0,1,0,0))
>
> ftest(fullmodel,L2)

      F     df1     df2 p-value
115.163   2.000  95.000   0.000

>
> # Test Country controlling for length and weight with full-reduced model
> reducedmodel = lm(lper100k ~ weight+length)
> anova(reducedmodel,fullmodel)

Analysis of Variance Table

Model 1: lper100k ~ weight + length
Model 2: lper100k ~ weight + length + Cntry
  Res.Df    RSS Df Sum of Sq      F   Pr(>F)
1     97 315.64
2     95 275.61  2    40.035 6.8999 0.001592 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```
