STA 302 Formulas

$M_y(t) = E(e^{yt})$

$M_{ay}(t) = M_y(at)$

$M_{y+a}(t) = e^{at}M_y(t)$

$M_{\sum_{i=1}^n y_i}(t) = \prod_{i=1}^n M_{y_i}(t)$

$y \sim N(\mu, \sigma^2)$ means $M_y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$W \sim \chi^2(\nu)$ means $M_W(t) = (1-2t)^{-\nu/2}$

If $W_1, \ldots, W_n \stackrel{ind}{\sim} \chi^2(\nu_i)$, then $\sum_{i=1}^n W_i \sim \chi^2(\sum_{i=1}^n \nu_i)$

If $Z \sim N(0,1)$ then $Z^2 \sim \chi^2(1)$

If $W = W_1 + W_2$ with W_1 and W_2 independent, $W \sim \chi^2(\nu_1 + \nu_2)$, $W_2 \sim \chi^2(\nu_2)$ then $W_1 \sim \chi^2(\nu_1)$

Columns of $\mathbf{A}$ *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{Av} = \mathbf{0}$.

Columns of $\mathbf{A}$ *linearly independent* means that $\mathbf{Av} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

$\mathbf{A}$ *positive definite* means $\mathbf{v}'\mathbf{Av} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$\boldsymbol{\Sigma} = \mathbf{CDC}'$

$\boldsymbol{\Sigma}^{-1} = \mathbf{CD}^{-1}\mathbf{C}'$

$\boldsymbol{\Sigma}^{1/2} = \mathbf{CD}^{1/2}\mathbf{C}'$

$\boldsymbol{\Sigma}^{-1/2} = \mathbf{CD}^{-1/2}\mathbf{C}'$

$cov(\mathbf{y}) = E\left\{(\mathbf{y} - \boldsymbol{\mu}_y)(\mathbf{y} - \boldsymbol{\mu}_y)'\right\}$

$C(\mathbf{y}, \mathbf{t}) = E\left\{(\mathbf{y} - \boldsymbol{\mu}_y)(\mathbf{t} - \boldsymbol{\mu}_t)'\right\}$

$cov(\mathbf{y}) = E\{\mathbf{yy}'\} - \boldsymbol{\mu}_y\boldsymbol{\mu}_y'$

$cov(\mathbf{Ay}) = \mathbf{A}cov(\mathbf{y})\mathbf{A}'$

$M_{\mathbf{y}}(\mathbf{t}) = E(e^{\mathbf{t}'\mathbf{y}})$

$M_{\mathbf{Ay}}(\mathbf{t}) = M_{\mathbf{y}}(\mathbf{A}'\mathbf{t})$

$M_{\mathbf{y}+\mathbf{c}}(\mathbf{t}) = e^{\mathbf{t}'\mathbf{c}}M_{\mathbf{y}}(\mathbf{t})$

$\mathbf{y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means $M_{\mathbf{y}}(\mathbf{t}) = e^{\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\boldsymbol{\Sigma}\mathbf{t}}$

$\mathbf{y}_1$ and $\mathbf{y}_2$ are independent if and only if $M_{(\mathbf{y}_1, \mathbf{y}_2)}(\mathbf{t}_1, \mathbf{t}_2) = M_{\mathbf{y}_1}(\mathbf{t}_1)M_{\mathbf{y}_2}(\mathbf{t}_2)$

If $\mathbf{y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\mathbf{Ay} \sim N_q(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}')$,

and $W = (\mathbf{y} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{y} - \boldsymbol{\mu}) \sim \chi^2(p)$

$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \epsilon_i$

$\epsilon_1, \ldots, \epsilon_n$ independent $N(0, \sigma^2)$

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ with $\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2\mathbf{I}_n)$

$\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \sim N_{k+1}(\boldsymbol{\beta}, \sigma^2(\mathbf{X}'\mathbf{X})^{-1})$

$\widehat{\mathbf{y}} = \mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{Hy}$, where $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$

$\widehat{\boldsymbol{\epsilon}} = \mathbf{y} - \widehat{\mathbf{y}} = (\mathbf{I} - \mathbf{H})\mathbf{y}$

$\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\epsilon}}$ are independent under normality.

$\frac{SSE}{\sigma^2} = \frac{\widehat{\boldsymbol{\epsilon}}'\widehat{\boldsymbol{\epsilon}}}{\sigma^2} \sim \chi^2(n-k-1)$

$\sum_{i=1}^n (y_i - \overline{y})^2 = \sum_{i=1}^n (y_i - \widehat{y}_i)^2 + \sum_{i=1}^n (\widehat{y}_i - \overline{y})^2$

$SST = SSE + SSR$ and $R^2 = \frac{SSR}{SST}$

$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$

$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2)$

$T = \frac{\mathbf{a}'\widehat{\boldsymbol{\beta}} - \mathbf{a}'\boldsymbol{\beta}}{\sqrt{MSE\,\mathbf{a}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}}} \sim t(n-k-1)$

$T = \frac{y_0 - \mathbf{x}_0'\widehat{\boldsymbol{\beta}}}{\sqrt{MSE\,(1 + \mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0)}} \sim t(n-k-1)$

$F = \frac{(\mathbf{C}\widehat{\boldsymbol{\beta}} - \mathbf{t})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\boldsymbol{\beta}} - \mathbf{t})}{q\,MSE} = \frac{SSR(full) - SSR(reduced)}{q\,MSE} \sim F(q, n-k-1)$, where $MSE = \frac{SSE}{n-k-1}$

$F = \left(\frac{p}{1-p}\right)\left(\frac{n-k-1}{q}\right) \Leftrightarrow p = \frac{qF}{qF+n-k-1}$, where $p = \frac{R^2(full) - R^2(reduced)}{1 - R^2(reduced)}$

$r_{xy} = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2}\sqrt{\sum_{i=1}^n (y_i - \overline{y})^2}}$
