

STA 302 Formulas

$$M_y(t) = E(e^{yt})$$

$$M_{ay}(t) = M_y(at)$$

$$M_{y+a}(t) = e^{at} M_y(t)$$

$$M_{\sum_{i=1}^n y_i}(t) = \prod_{i=1}^n M_{y_i}(t)$$

$$y \sim N(\mu, \sigma^2) \text{ means } M_y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$W \sim \chi^2(\nu) \text{ means } M_W(t) = (1 - 2t)^{-\nu/2}$$

$$\text{If } W_1, \dots, W_n \stackrel{\text{ind}}{\sim} \chi^2(\nu_i), \text{ then } \sum_{i=1}^n W_i \sim \chi^2(\sum_{i=1}^n \nu_i)$$

$$\text{If } Z \sim N(0, 1) \text{ then } Z^2 \sim \chi^2(1)$$

$$\text{If } W = W_1 + W_2 \text{ with } W_1 \text{ and } W_2 \text{ independent, } W \sim \chi^2(\nu_1 + \nu_2), W_2 \sim \chi^2(\nu_2) \text{ then } W_1 \sim \chi^2(\nu_1)$$

Columns of \mathbf{A} *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{A}\mathbf{v} = \mathbf{0}$.

Columns of \mathbf{A} *linearly independent* means that $\mathbf{A}\mathbf{v} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

\mathbf{A} *positive definite* means $\mathbf{v}'\mathbf{A}\mathbf{v} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$$\Sigma = \mathbf{C}\mathbf{D}\mathbf{C}'$$

$$\Sigma^{-1} = \mathbf{C}\mathbf{D}^{-1}\mathbf{C}'$$

$$\Sigma^{1/2} = \mathbf{C}\mathbf{D}^{1/2}\mathbf{C}'$$

$$\Sigma^{-1/2} = \mathbf{C}\mathbf{D}^{-1/2}\mathbf{C}'$$

$$\text{cov}(\mathbf{y}) = E\{(\mathbf{y} - \boldsymbol{\mu}_y)(\mathbf{y} - \boldsymbol{\mu}_y)'\}$$

$$C(\mathbf{y}, \mathbf{t}) = E\{(\mathbf{y} - \boldsymbol{\mu}_y)(\mathbf{t} - \boldsymbol{\mu}_t)'\}$$

$$\text{cov}(\mathbf{y}) = E\{\mathbf{y}\mathbf{y}'\} - \boldsymbol{\mu}_y\boldsymbol{\mu}_y'$$

$$\text{cov}(\mathbf{A}\mathbf{y}) = \mathbf{A}\text{cov}(\mathbf{y})\mathbf{A}'$$

$$M_{\mathbf{y}}(\mathbf{t}) = E(e^{\mathbf{t}'\mathbf{y}})$$

$$M_{\mathbf{A}\mathbf{y}}(\mathbf{t}) = M_{\mathbf{y}}(\mathbf{A}'\mathbf{t})$$

$$M_{\mathbf{y}+\mathbf{c}}(\mathbf{t}) = e^{\mathbf{t}'\mathbf{c}} M_{\mathbf{y}}(\mathbf{t})$$

$$\mathbf{y} \sim N_p(\boldsymbol{\mu}, \Sigma) \text{ means } M_{\mathbf{y}}(\mathbf{t}) = e^{\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\Sigma\mathbf{t}}$$

$$\mathbf{y}_1 \text{ and } \mathbf{y}_2 \text{ are independent if and only if } M_{(\mathbf{y}_1, \mathbf{y}_2)}(\mathbf{t}_1, \mathbf{t}_2) = M_{\mathbf{y}_1}(\mathbf{t}_1)M_{\mathbf{y}_2}(\mathbf{t}_2)$$

$$\text{If } \mathbf{y} \sim N_p(\boldsymbol{\mu}, \Sigma), \text{ then } \mathbf{A}\mathbf{y} \sim N_q(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\Sigma\mathbf{A}'),$$

$$\text{and } W = (\mathbf{y} - \boldsymbol{\mu})'\Sigma^{-1}(\mathbf{y} - \boldsymbol{\mu}) \sim \chi^2(p)$$

$$y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_k x_{ik} + \epsilon_i$$

$$\epsilon_1, \dots, \epsilon_n \text{ independent } N(0, \sigma^2)$$

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \text{ with } \boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2\mathbf{I}_n)$$

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \sim N_{k+1}(\boldsymbol{\beta}, \sigma^2(\mathbf{X}'\mathbf{X})^{-1})$$

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{H}\mathbf{y}, \text{ where } \mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$$

$$\hat{\boldsymbol{\epsilon}} = \mathbf{y} - \hat{\mathbf{y}} = (\mathbf{I} - \mathbf{H})\mathbf{y}$$

$\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\epsilon}}$ are independent under normality.

$$\frac{SSE}{\sigma^2} = \frac{\hat{\boldsymbol{\epsilon}}'\hat{\boldsymbol{\epsilon}}}{\sigma^2} \sim \chi^2(n - k - 1)$$

$$\sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$$

$$SST = SSE + SSR \text{ and } R^2 = \frac{SSR}{SST}$$

$$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$$

$$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2)$$

$$T = \frac{\mathbf{a}'\hat{\boldsymbol{\beta}} - \mathbf{a}'\boldsymbol{\beta}}{\sqrt{MSE \mathbf{a}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}}} \sim t(n - k - 1)$$

$$T = \frac{y_0 - \mathbf{x}'_0\hat{\boldsymbol{\beta}}}{\sqrt{MSE(1 + \mathbf{x}'_0(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0)}} \sim t(n - k - 1)$$

$$F = \frac{(\mathbf{C}\hat{\boldsymbol{\beta}} - \mathbf{t})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\hat{\boldsymbol{\beta}} - \mathbf{t})}{q \text{ MSE}} = \frac{SSR(\text{full}) - SSR(\text{reduced})}{q \text{ MSE}} \sim F(q, n - k - 1), \text{ where } MSE = \frac{SSE}{n - k - 1}$$

$$F = \left(\frac{p}{1-p}\right)\left(\frac{n-k-1}{q}\right) \Leftrightarrow p = \frac{qF}{qF + n - k - 1},$$

$$\text{where } p = \frac{R^2(\text{full}) - R^2(\text{reduced})}{1 - R^2(\text{reduced})}$$

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

This formula sheet was prepared by [Jerry Brunner](#), Department of Statistics, University of Toronto. It is licensed under a [Creative Commons Attribution - ShareAlike 3.0 Unported License](#). Use any part of it as you like and share the result freely. The \LaTeX source code is available from the course website:

<http://www.utstat.toronto.edu/~brunner/oldclass/302f15>