

Assignment 9, Question One

First, note that $\hat{\beta}_1 = r \sqrt{\frac{\sum (y_i - \bar{y})^2}{\sum (x_i - \bar{x})^2}} = r \frac{\sqrt{SST}}{\sqrt{\sum (x_i - \bar{x})^2}}$

It's easier to start with R^2 :

$$R^2 = \frac{SSR}{SST} = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{SST}$$

substitute
 $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$$= \frac{\sum_{i=1}^n (\hat{\beta}_0 + \hat{\beta}_1 x_i - \hat{\beta}_0 - \hat{\beta}_1 \bar{x})^2}{SST} = \frac{(\hat{\beta}_1^2 \sum_{i=1}^n (x_i - \bar{x})^2)}{SST}$$

$$= r^2 \frac{SST}{\sum_{i=1}^n (x_i - \bar{x})^2} \cdot \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{SST} = r^2$$