

Name Jenny

Student Number _____

STA 302 f2014 Quiz 7A

1. Suppose that in the linear model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ where $\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2 \mathbf{I}_n)$, it happens just by chance that $\mathbf{Y} = \mathbf{X}\mathbf{b}$ for some $\mathbf{b} \in \mathbb{R}^{k+1}$. That is, \mathbf{Y} happens to fall into the subspace spanned by the columns of \mathbf{X} . When you fit the regression model, some strange things happen.

- (a) What is $\hat{\boldsymbol{\beta}}$? Show your work and ~~simplify~~.

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{X}\mathbf{b} = \mathbf{b}$$

- (b) What is $\hat{\mathbf{Y}}$? ~~Show your work and simplify~~

$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}\mathbf{b}$$

- (c) What is $\hat{\boldsymbol{\epsilon}}$? Show your work and simplify.

$$\hat{\boldsymbol{\epsilon}} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}\mathbf{b} - \mathbf{X}\mathbf{b} = \mathbf{0}$$

- (d) What is MSE ? Show your work

$$MSE = \frac{SSE}{n-k-1} = \frac{\hat{\boldsymbol{\epsilon}}'\hat{\boldsymbol{\epsilon}}}{n-k-1} = 0$$

- (e) What is the probability that this will happen if the model is correct? The answer is a number. Why?

Zero, because the distribution of \mathbf{Y} is continuous and the space spanned by the columns of \mathbf{X} is of lower dimension.

(Please be forgiving with the reason.)

2. (2 points) Holding all other independent variables constant at fixed values, estimate the amount by which the crime rate changes when the percent of residents in a census tract who are 65 or older is increased by one. The answer is a number. On your printout, circle the number and write "Question 2" beside it.

0.0858

3. (3 points) A confidence interval is the estimate plus or minus a margin of error. Give the 95% margin of error for the estimate in the last question. Your answer is a number. Show a little work. In case you did not print the critical value, it is 1.978.

$$0.4465 * 1.978 = 0.883$$

Please attach your printout to the quiz paper. Make sure your name is on the printout.

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STA 302 f2014 Quiz 7AB

1. Suppose you have a random sample from a normal distribution. If someone randomly sampled another observation from this population and asked you to guess what it was, there is no doubt you would give the sample mean. But what if you were asked for a prediction *interval*?

Accordingly, let $Y_1, \dots, Y_n \stackrel{i.i.d.}{\sim} N(\mu, \sigma^2)$, and let Y_0 denote another observation independently sampled from this distribution. You already know that \bar{Y} is normally distributed, $\frac{1}{\sigma^2} \sum_{i=1}^n (Y_i - \bar{Y})^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, and that \bar{Y} and S^2 are independent.

- (a) What is the distribution of $Y_0 - \bar{Y}$? You don't have to show any work; just write down the answer.

$$Y_0 - \bar{Y} \sim N(0, \sigma^2 + \frac{\sigma^2}{n})$$

- (b) Now standardize, obtaining a standard normal random variable that could be called Z .

$$Z = \frac{Y_0 - \bar{Y}}{\sqrt{\sigma^2(1 + 1/n)}}$$

- (c) Look at the definition of the t distribution on the formula sheet, and divide Z by the appropriate quantity. Call the result T . Simplify a little.

$$T = \frac{Y_0 - \bar{Y} / \sqrt{\sigma^2(1 + 1/n)}}{\sqrt{\frac{(n-1)S^2}{\sigma^2} / (n-1)}} = \frac{Y_0 - \bar{Y}}{S\sqrt{1 + 1/n}}$$

- (d) If you have done this right, T has a t distribution. What are the degrees of freedom?

$$n-1$$

- (e) How do you know numerator and denominator are independent?

Y_0 and \bar{Y} are both independent of S^2

(no more explanation is needed)

2. (2 points) Holding all other independent variables constant at fixed values, estimate the amount by which the crime rate changes when the percent of residents in a census tract who live in urban areas is increased by one. The answer is a number. On your printout, circle the number and write "Question 2" beside it.

$$0.149$$

3. (3 points) A confidence interval is the estimate plus or minus a margin of error. Give the 95% margin of error for the estimate in the last question. Your answer is a number. Show a little work. In case you did not print the critical value, it is 1.978.

$$0.0638 \times 1.978 = 0.1266$$

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