Name Jerry

Student Number

STA 302 f2014 Quiz 4A

The $n \times 1$ matrix $\widehat{\mathbf{Y}} = \mathbf{X}\widehat{\boldsymbol{\beta}}$ is a point in $\mathbb{R}^n$. Geometrically, it is the *projection* (shadow) of $\mathbf{Y}$ onto the subset of vectors spanned by the columns of the $\mathbf{X}$ matrix. This means the vector of differences $\widehat{\boldsymbol{\epsilon}} = \mathbf{Y} - \widehat{\mathbf{Y}}$ should be perpendicular (at right angles) to each and every vector of the form $\mathbf{Xb}$, where $\mathbf{b} \in \mathbb{R}^{k+1}$. Prove it, by calculating the inner product $(\mathbf{Xb})'\widehat{\boldsymbol{\epsilon}}$ for general $\mathbf{b}$. You don't have to fill the page. You have a lot more room than you need.

$$(Xb)'\hat{\varepsilon} = b'X'(Y - X\hat{\beta})$$

$$= b'X'Y - b'\underbrace{X'X(X'X)^{-1}}_{I}X'Y = b'X'Y - b'X'Y = 0$$

Name Jenny

Student Number

STA 302 f2014 Quiz 4B

1. (5 points) In Part (vii) of homework question (2g), you were asked to compute a simple regression with R. What is the slope of the least squares line? Copy the answer into the space below. Attach the R printout, and **Circle the answer on the printout,** Make sure your name is on the printout. and write Question 1 beside the answer.

$\hat{\beta}_1 = -1.41$

2. In simple regression through the origin, the model is $Y_i = \beta x_i + \epsilon_i$. ~~Writing~~ Expressing this in matrix form as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$,

   (a) (1 point) What is the $\mathbf{X}$ matrix?

$$\underset{\sim}{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

   (b) (1 point) What is $\mathbf{X}'\mathbf{X}$?

$$\sum_{i=1}^{n} x_i^2$$

   (c) (1 point) What is $\mathbf{X}'\mathbf{Y}$?

$$\sum_{i=1}^{n} x_i y_i$$

   (d) (2 points) What is $\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$?

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$$