STA 302 Formulas

$M_Y(t) = E(e^{Yt})$

$M_{aY}(t) = M_Y(at)$

$M_{Y+a}(t) = e^{at}M_Y(t)$

$M_{\sum_{i=1}^n Y_i}(t) = \prod_{i=1}^n M_{Y_i}(t)$

$Y \sim N(\mu, \sigma^2)$ means $M_Y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$W \sim \chi^2(\nu)$ means $M_W(t) = (1-2t)^{-\nu/2}$

If $W_1, \ldots, W_n \overset{ind}{\sim} \chi^2(\nu_i)$, then $\sum_{i=1}^n W_i \sim \chi^2(\sum_{i=1}^n \nu_i)$

If $Z \sim N(0,1)$ then $Z^2 \sim \chi^2(1)$

If $W = W_1 + W_2$ with W_1 and W_2 independent, $W \sim \chi^2(\nu_1+\nu_2)$, $W_2 \sim \chi^2(\nu_2)$ then $W_1 \sim \chi^2(\nu_1)$

Columns of $\mathbf{A}$ *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{Av} = \mathbf{0}$.

Columns of $\mathbf{A}$ *linearly independent* means that $\mathbf{Av} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

$\mathbf{A}$ *positive definite* means $\mathbf{v}'\mathbf{Av} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$\boldsymbol{\Sigma} = \mathbf{CDC}'$

$\boldsymbol{\Sigma}^{-1} = \mathbf{CD}^{-1}\mathbf{C}'$

$\boldsymbol{\Sigma}^{1/2} = \mathbf{CD}^{1/2}\mathbf{C}'$

$\boldsymbol{\Sigma}^{-1/2} = \mathbf{CD}^{-1/2}\mathbf{C}'$

$cov(\mathbf{Y}) = E\left\{(\mathbf{Y}-\boldsymbol{\mu}_y)(\mathbf{Y}-\boldsymbol{\mu}_y)'\right\}$

$C(\mathbf{Y},\mathbf{T}) = E\left\{(\mathbf{Y}-\boldsymbol{\mu}_y)(\mathbf{T}-\boldsymbol{\mu}_t)'\right\}$

$cov(\mathbf{Y}) = E\{\mathbf{YY}'\} - \boldsymbol{\mu}_y\boldsymbol{\mu}_y'$

$cov(\mathbf{AY}) = \mathbf{A}cov(\mathbf{Y})\mathbf{A}'$

$M_{\mathbf{Y}}(\mathbf{t}) = E(e^{\mathbf{t}'\mathbf{Y}})$

$M_{\mathbf{AY}}(\mathbf{t}) = M_{\mathbf{Y}}(\mathbf{A}'\mathbf{t})$

$M_{\mathbf{Y}+\mathbf{c}}(\mathbf{t}) = e^{\mathbf{t}'\mathbf{c}}M_{\mathbf{Y}}(\mathbf{t})$

$\mathbf{Y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means $M_{\mathbf{Y}}(\mathbf{t}) = e^{\mathbf{t}'\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}'\boldsymbol{\Sigma}\mathbf{t}}$

$\mathbf{Y}_1$ and $\mathbf{Y}_2$ are independent if and only if $M_{(\mathbf{Y}_1,\mathbf{Y}_2)}(\mathbf{t}_1,\mathbf{t}_2) = M_{\mathbf{Y}_1}(\mathbf{t}_1)M_{\mathbf{Y}_2}(\mathbf{t}_2)$

If $\mathbf{Y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\mathbf{AY} \sim N_q(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}')$,

and $W = (\mathbf{Y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{Y}-\boldsymbol{\mu}) \sim \chi^2(p)$

$Y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \epsilon_i$

$\epsilon_1, \ldots, \epsilon_n$ independent $N(0, \sigma^2)$

$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ with $\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2\mathbf{I}_n)$

$\widehat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \sim N_{k+1}(\boldsymbol{\beta}, \sigma^2(\mathbf{X}'\mathbf{X})^{-1})$

$\widehat{\mathbf{Y}} = \mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{HY}$, where $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$

$\widehat{\boldsymbol{\epsilon}} = \mathbf{Y} - \widehat{\mathbf{Y}} = (\mathbf{I}-\mathbf{H})\mathbf{Y}$

$\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\epsilon}}$ are independent under normality.

$\frac{SSE}{\sigma^2} = \frac{\widehat{\boldsymbol{\epsilon}}'\widehat{\boldsymbol{\epsilon}}}{\sigma^2} \sim \chi^2(n-k-1)$

$\sum_{i=1}^n (Y_i - \overline{Y})^2 = \sum_{i=1}^n (Y_i - \widehat{Y}_i)^2 + \sum_{i=1}^n (\widehat{Y}_i - \overline{Y})^2$

$SST = SSE + SSR$ and $R^2 = \frac{SSR}{SST}$

$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$

$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2)$

$T = \frac{\mathbf{a}'\widehat{\boldsymbol{\beta}} - \mathbf{a}'\boldsymbol{\beta}}{\sqrt{MSE\,\mathbf{a}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{a}}} \sim t(n-k-1)$

$T = \frac{Y_0 - \mathbf{x}_0'\widehat{\boldsymbol{\beta}}}{\sqrt{MSE\,(1+\mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0)}} \sim t(n-k-1)$

$F = \frac{(\mathbf{C}\widehat{\boldsymbol{\beta}}-\mathbf{t})'(\mathbf{C}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\boldsymbol{\beta}}-\mathbf{t})}{q\,MSE} = \frac{SSR - SSR(reduced)}{q\,MSE} \sim F(q, n-k-1)$, where $MSE = \frac{SSE}{n-k-1}$

$F = \left(\frac{a}{1-a}\right)\left(\frac{n-k-1}{q}\right) \Leftrightarrow a = \frac{qF}{n-k-1+qF}$, where $a = \frac{R^2 - R^2(reduced)}{1-R^2(reduced)}$

$r_{xy} = \frac{\sum_{i=1}^n (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^n (X_i-\overline{X})^2}\sqrt{\sum_{i=1}^n (Y_i-\overline{Y})^2}}$

$\log\left(\frac{\pi_i}{1-\pi_i}\right) = \beta_0 + \beta_1 x_{i,1} + \ldots + \beta_k x_{i,k}$

$\pi_i = \frac{e^{\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_k x_{i,k}}}{1 + e^{\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_k x_{i,k}}}$