Family (Last) Name ____________________

Given (First) Name Jerry

Student Number ____________________

STA 302s13 Quiz 3A

1. (4 points) For this question, recall that the spectral decomposition of a symmetric matrix $\mathbf{A}$ is $\mathbf{A} = \mathbf{CDC}'$. Let $\mathbf{A}$ be a square symmetric matrix with eigenvalues that are all strictly positive.

   (a) Suppose $\mathbf{A}$ is 3×3, with eigenvalues λ_1, λ_2 and λ_3. What is $\mathbf{D}^{-1}$? Just write down the answer. No proof is required.

$$D^{-1} = \begin{pmatrix} 1/\lambda_1 & 0 & 0 \\ 0 & 1/\lambda_2 & 0 \\ 0 & 0 & 1/\lambda_3 \end{pmatrix}.$$

   (b) Prove $\mathbf{A}^{-1} = \mathbf{CD}^{-1}\mathbf{C}'$. You have two things to show.

(1) $$CD^{-1}C'A = CD^{-1}\underbrace{C'C}_{I}DC' = C\underbrace{D^{-1}D}_{I}C'$$
$$= CC' = I$$

(2) $$ACD^{-1}C' = CD\underbrace{C'C}_{I}D^{-1}C' = C\underbrace{DD^{-1}}_{I}C'$$
$$= CC' = I$$

done

expected value μ_x and

2. (4 points) If the $p \times 1$ random vector $\mathbf{X}$ has variance-covariance matrix $\boldsymbol{\Sigma}$ and $\mathbf{A}$ is an $m \times p$ matrix of constants, prove that the variance-covariance matrix of $\mathbf{AX}$ is $\mathbf{A\Sigma A'}$. Start with the definition of a variance-covariance matrix:

$$cov(\mathbf{Z}) = E\{(\mathbf{Z} - \boldsymbol{\mu}_z)(\mathbf{Z} - \boldsymbol{\mu}_z)'\}.$$

$E(AX) = A\mu_x$, so

$$
\begin{aligned}
cov(AX) &= E\{(AX - A\mu_x)(AX - A\mu_x)'\} \\
&= E\{A(X-\mu_x)(A(X-\mu_x))'\} \\
&= E\{A(X-\mu_x)(X-\mu_x)'A'\} \\
&= A\,E\{(X-\mu_x)(X-\mu_x)'\}A' \\
&= A\Sigma A'
\end{aligned}
$$

3. (2 points) Attach the R output for your answer to Homework Question Two: That's Question 2.77 in the text. Make sure your name is on the printout.