Indicator functions: This notation is not in the text!

Let A be a set of real numbers. Then the **indicator function** for A is defined by

$$I_A(x) = I\{x \in A\} = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

Ex. $I\{x \geq 0\} = I_{[0,\infty)}(x)$ $\quad$ $I\{x=1,2,3\} = I_{\{1,2,3\}}(x)$

$I\{a < x \leq b\} = I_{(a,b]}(x)$ $\quad$ $I\{x=0,1, \ldots\} = I_{\{0,1, \ldots\}}(x)$

Here are three important properties of indicator functions:

- If g(x) is a real valued function, $g(x)\, I_A(x) = \begin{cases} g(x) & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$

- $I_{A \cap B}(x) = I_A(x)\, I_B(x)$, and

- $P(A) = \int_{-\infty}^{\infty} I(x \in A) f(x)\, dx = E(I\{X \in A\})$

Def. The **support** of a random variable is the set of x values for which $f(x) > 0$.

In this class, probability distributions and probability density functions will always be defined for all real x, and will include indicators for their support.

For example, where the book might write $\quad f(x) = \begin{cases} \frac{x}{6} & \text{for } x = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$

we will write $\quad f(x) = \frac{x}{6}\, I\{x = 1,2,3\}.$

And the gamma density may be written $\quad f(x) = \frac{1}{\beta^\alpha\, \Gamma(\alpha)}\, e^{-x/\beta}\, x^{\alpha-1}\, I(x>0).$