

# STA 261 Formulas

Distribution	$f(x)$	$M(t)$	$E(X)$	$Var(X)$
Bernoulli	$\theta^x(1-\theta)^{1-x}I(x=0,1)$	$\theta e^t + 1 - \theta$	$\theta$	$\theta(1-\theta)$
Binomial	$\binom{n}{x}\theta^x(1-\theta)^{n-x}I(x=0,\dots,n)$	$(\theta e^t + 1 - \theta)^n$	$n\theta$	$n\theta(1-\theta)$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}I(x=0,1,\dots)$	$e^{\lambda(e^t-1)}$	$\lambda$	$\lambda$
Geometric	$\theta(1-\theta)^{x-1}I(x=1,2,\dots)$	$\theta(e^{-t} + \theta - 1)^{-1}$	$\frac{1}{\theta}$	$\frac{1-\theta}{\theta^2}$
Uniform	$\frac{1}{\beta-\alpha}I(\alpha \leq x \leq \beta)$	$\frac{e^{\beta t} - e^{\alpha t}}{t(\beta-\alpha)}$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$
Exponential	$\frac{1}{\theta}e^{-x/\theta}I(x > 0)$	$(1 - \theta t)^{-1}$	$\theta$	$\theta^2$
Gamma	$\frac{1}{\beta^\alpha \Gamma(\alpha)}e^{-x/\beta}x^{\alpha-1}I(x > 0)$	$(1 - \beta t)^{-\alpha}$	$\alpha\beta$	$\alpha\beta^2$
Chi-square	$\frac{1}{2^{\nu/2}\Gamma(\nu/2)}e^{-x/2}x^{\nu/2-1}I(x > 0)$	$(1 - 2t)^{-\nu/2}$	$\nu$	$2\nu$
Normal	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$	$\mu$	$\sigma^2$

$T_n \xrightarrow{P} T$  means for every  $c > 0$ ,  $\lim_{n \rightarrow \infty} P\{|T_n - T| < c\} = 1$   
 If  $Z \sim N(0, 1)$  and  $Y \sim \chi^2(\nu)$  are independent, then  $T = \frac{Z}{\sqrt{Y/\nu}} \sim t(\nu)$ .

If  $U \sim \chi^2(\nu_1)$  and  $V \sim \chi^2(\nu_2)$  are independent, then  $F = \frac{U/\nu_1}{V/\nu_2} \sim F(\nu_1, \nu_2)$ .

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \qquad \sum_{k=r}^{\infty} a^k = \frac{a^r}{1-a} \text{ for } 0 < a < 1$$

$$E(g(X)) \geq a Pr\{g(X) \geq a\} \qquad Pr\{|X - \mu| \geq k\sigma\} \leq \frac{1}{k^2}$$

$$F_{Y_1}(y) = 1 - [1 - F(y)]^n \qquad f_{Y_1}(y) = n[1 - F(y)]^{n-1}f(y)$$

$$F_{Y_n}(y) = [F(y)]^n \qquad f_{Y_n}(y) = n[F(y)]^{n-1}f(y)$$

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \frac{1}{n-1} \left( \sum_{i=1}^n X_i^2 - n\bar{X}^2 \right) \qquad \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$Z = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\hat{\sigma}} \qquad T = \frac{\sqrt{n}(\bar{X}_n - \mu)}{S} \sim t(n-1) \qquad \bar{x} \pm z_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}} \text{ or } \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$Z = \frac{(\bar{X}_{n_1} - \bar{Y}_{n_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\hat{\sigma}_1^2}{n_1} + \frac{\hat{\sigma}_2^2}{n_2}}} \qquad T = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2) \qquad S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}}$$

$$Z = \frac{\bar{X} - \theta_0}{\sqrt{\frac{\theta_0(1-\theta_0)}{n}}} \qquad F = \frac{S_1^2}{S_2^2} \sim F(n_1 - 1, n_2 - 1) \qquad C = \{\mathbf{x} : \frac{f(\mathbf{x}; \theta_0)}{f(\mathbf{x}; \theta_1)} < k\}$$

$$\lambda(\mathbf{x}) = \frac{\max_{\theta \in \omega} f(\mathbf{x}; \theta)}{\max_{\theta \in \Omega} f(\mathbf{x}; \theta)} = \frac{f(\mathbf{x}; \hat{\theta})}{f(\mathbf{x}; \hat{\theta})} \qquad C = \{\mathbf{x} : \lambda(\mathbf{x}) \leq k\}, 0 < k < 1 \qquad G = -2 \ln \lambda(\mathbf{X})$$