STA 261s2006 Assignment 10

Do this assignment in preparation for the quiz on Wednesday, March 22nd. The questions are practice for the quiz, and are not to be handed in.

1. Please read Sections 12.1 and 12.2 in the text. Do exercises 12.1, 12.5, 12.6 (my answers are 0.777 and 0.549), 12.7, 12.35, and 12.39.

2. $X_1, \ldots, X_{n_1}$ be a random sample. For each of the distributions below, give the parameter space Ω.

   (a) Bernoulli

   (b) Binomial(k, θ) with k known

   (c) Binomial(k, θ) with k unknown

   (d) Poisson

   (e) Geometric

   (f) Uniform(α, β)

   (g) Exponential

   (h) Gamma

   (i) Normal

3. Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with expected value μ and variance σ^2.

   (a) We will test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ using the critical region

   $$C = \{(x_1, \ldots, x_n) : \left|\frac{\overline{x} - \mu_0}{s/\sqrt{n}}\right| > t_{\alpha/2,n-1}\}$$

   This is called a *one-sample t-test.* If the observations are differences (like pretest versus post-test) and $\mu_0 = 0$, it is called a *matched t-test.*

   i. What is ω? Is it simple or composite?

   ii. What is ω'? Is it simple or composite?

   iii. What is the size of the test? Prove your answer. Start with the distribution of $T = \frac{\overline{X}-\mu_0}{S/\sqrt{n}}$ when $H_0 : \mu = \mu_0$ is true (use results from the section on confidence intervals).

   iv. Show that H_0 is rejected if and only if the $(1-\alpha)100\%$ confidence interval for μ does *not* include μ_0. It is easiest to start by writing the set of $(x_1, \ldots, x_n)$ such that μ_0 is in the confidence interval, and then work on it until it becomes C^c.

   v. Suppose the data are not really normal, but the sample size is large. Does the test still have the same approximate size? Answer Yes or No and explain why.

4. Still for the single sample from a normal distribution, we will test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ using the critical region

$$C = \{(x_1, \ldots, x_n) : \frac{(n-1)s^2}{\sigma_0^2} > \chi^2_{\alpha/2, n-1} \text{ or } \frac{(n-1)s^2}{\sigma_0^2} < \chi^2_{1-\alpha/2, n-1}\}$$

   (a) What is ω? Is it simple or composite?

   (b) What is ω'? Is it simple or composite?

   (c) What is the size of the test? Prove your answer starting with the distribution of $\frac{(n-1)S^2}{\sigma_0^2}$ when $H_0 : \sigma^2 = \sigma_0^2$ is true (use results from the section on confidence intervals).

5. Let $X_1, \ldots, X_{n_1}$ be a random sample from a $N(\mu_1, \sigma^2)$ distribution, and let $Y_1, \ldots, Y_{n_2}$ be a random sample from a $N(\mu_2, \sigma^2)$ distribution. These are *independent* random samples, meaning that the X and Y values are independent.

   (a) What is ω? Is it simple or composite?

   (b) What is ω'? Is it simple or composite?

   (c) Using the facts that $\overline{X}$ and S^2 are independent and $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ for each sample, give a size α critical region for testing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$. Hint: Look at the corresponding confidence interval from the last assignment. Your answer is called a *two-sample t-test*, or an *independent t-test*.

6. Let $X_1, \ldots, X_{n_1}$ be a random sample from a (possibly) non-normal distribution with mean μ_1 and variance σ_1^2), and let $Y_1, \ldots, Y_{n_2}$ be a random sample from a (possibly) non-normal distribution with mean μ_2 and variance σ_2^2). These are *independent* random samples, meaning that the data are independent between samples as well as within samples. We are interested in testing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$. Find the constant k so that the following critical region will have, for large n_1 and large n_2, a size of approximately α.

$$C = \left\{ (x_1, \ldots, x_n) : \left| \frac{\overline{x}_{n_1} - \overline{y}_{n_2}}{\sqrt{\frac{\widehat{\sigma}_1^2}{n_1} + \frac{\widehat{\sigma}_2^2}{n_2}}} \right| > k \right\},$$

   where $\widehat{\sigma}_1^2$ and $\widehat{\sigma}_2^2$ are consistent estimators of σ_1^2 and σ_2^2, respectively.

7. Of a random sample of 150 Special Needs students in the Toronto District School Board, 19 were in regular classes, and the rest were in Special Education classes. Of a random sample of 200 Special Needs students in the Toronto Separate School Board, 48 were in regular classes, and the rest were in Special Education classes. Test for difference between the proportions of Special Needs students in regular classes in the two school boards. Use $\alpha = 0.01$. What do you conclude? Of course you should use the test from the last question.

8. Let $X_1, \ldots, X_{n_1}$ be a random sample from an exponential distribution with parameter θ. Give a size α critical region for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Your answer should use chi-square critical values.

9. Let $X_1, \ldots, X_{n_1}$ be a random sample from a uniform distribution on $(0, \theta]$. We will test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ using the critical region

$$C = \{(x_1, \ldots, x_n) : Y_n > \theta_0 \text{ or } Y_n < k\theta_0\},$$

where $k > 0$ is a constant.

(a) Find the constant k so that the test has size α.

(b) Suppose $n = 25$, $\theta_0 = 2$ and $\alpha = 0.05$. What is k? My answer (rounded) is 0.887.

10. Let $X_1, \ldots, X_{n_1}$ be a random sample from a *shifted exponential* distribution; that is, $f(x; \theta) = e^{-(x-\theta)} I(x > \theta)$ and $F(x; \theta) = (1 - e^{-(x-\theta)}) I(x > \theta)$, where θ is any real number. We will reject $H_0 : \theta = \theta_0$ in favor of $H_1 : \theta \neq \theta_0$ if either $Y_1 < \theta_0$ or $Y_1 > \theta_0 + k$, where k is a positive constant.

(a) Find the constant k so that the test has size α.

(b) Suppose $n = 25$, $\theta_0 = 2$ and $\alpha = 0.05$. What is k? My answer (rounded) is 0.1198.