STA 261s2006 Assignment 1

Do this assignment in preparation for the quiz on Wednesday, Jan. 11th. The questions are practice for the quiz, and are not to be handed in.

1. Let X_1 and X_2 be continuous random variables, and $Y = g(X_1, X_2)$. In this course, you may use the following Big Theorem like a definition:

$$E[Y] = E[g(X_1, X_2)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x_1, x_2) f(x_1, x_2) \, dx_1 dx_2.$$

   It extends to more than two random variables, and also to discrete random variables.

   (a) Letting a and b be constants, what is what is $E[aX + b]$? Show your work; use integrals.

   (b) Show $E[X + Y] = E[X] + E[Y]$. Use integrals. If you assume X and Y independent, you will lose at least half marks.

   (c) Let X and Y be independent. Show $E[XY] = E[X]E[Y]$. Use integrals. State clearly where you use the assumption of independence. Draw an arrow to the place in your calculation, and write "I use the assumption of independence here."

2. Let X and Y be independent, with $E[X] = \mu_x$ and $E[Y] = \mu_y$. The definition $Var(X) = E[(X - \mu_x)^2]$ is something you should know. Prove $Var[X + Y] = Var[X] + Var[Y]$. There is no need for integrals this time.

3. Prove Theorem 4.7 on p. 141.

4. Re-read Section 4.5. Do exercises 4.23, 4.33, 4.34, 4.35, 4.36, (Hint: Consider $t = 1/2$), 4.39.

5. Derive the moment-generating functions for the following distributions.

   (a) Poisson with parameter $\lambda > 0$

   (b) Gamma with parameters $\alpha > 0$ and $\beta > 0$.

   (c) Standard normal.

6. Re-read Section 7.2. There is value in problems like Example 7.3, where you have to integrate in two dimensions, but we are in a hurry and so you're not responsible for it. Do Exercises 7.1, 7.2 and 7.3. In each case, state the values of y for which the density of Y is non-zero. This is very important in terms of marks.

7. Let X be a random variable from a gamma distribution with parameters $\alpha > 0$ and $\beta > 0$. Let $Y = aX$, where the constant $a > 0$.

   (a) Find the density of Y. That is, find $f_Y(y)$.
   (b) For what values of y is $f_Y(y)$ greater than zero?
   (c) What happens if $a = 0$?

8. Let X be a random variable from a normal distribution with parameters μ and $\sigma^2 > 0$. Let $Y = aX + b$, where a and b are constants with $a \neq 0$.

   (a) Find the density of Y. That is, find $f_Y(y)$.
   (b) For what values of y is $f_Y(y)$ greater than zero?
   (c) What happens if $a = 0$?

9. Let X be a random variable from a uniform distribution with parameters $\alpha = 0$ and $\beta = 1$. Let $Y = -\ln(X)$.

   (a) Find the density of Y. That is, find $f_Y(y)$.
   (b) For what values of y is $f_Y(y)$ greater than zero?

10. Let X be a random variable from an exponential distribution with parameter $\theta > 0$. Let $Y = X + c$, where c is a constant.

   (a) Find the density of Y. That is, find $f_Y(y)$.
   (b) For what values of y is $f_Y(y)$ greater than zero?

11. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables from a continuous distribution with density $f(x)$ and cumulative distribution function $F(x)$. Let $Y = \max(X_1, \ldots, X_n)$.

   (a) Find the density of Y. That is, find $f_Y(y)$. Hint: Use the distribution function technque.
   (b) For what values of y is $f_Y(y)$ greater than zero?

12. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables from a continuous distribution with density $f(x)$ and cumulative distribution function $F(x)$. Let $Y = \min(X_1, \ldots, X_n)$.

   (a) Find the density of Y. That is, find $f_Y(y)$. Hint: Use the distribution function technque.
   (b) For what values of y is $f_Y(y)$ greater than zero?