Name Jerry

Student Number ____________

STA 260 S2020 Exam Practice 2

Let $X_1, \ldots, X_{n_1}$ be a random sample from a geometric distribution with parameter θ, and let the prior distribution of θ be uniform on the interval from zero to one.

1. (70 Points) Find the density of the posterior distribution, including the constant that makes it integrate to one. Name the distribution.

$$\pi(\theta \mid \underset{\sim}{x}) \propto \prod_{i=1}^{n} (1-\theta)^{x_i} \theta \, I(0 \le \theta \le 1) = (1-\theta)^{\Sigma x_i} \theta^n \, I(0 \le \theta \le 1)$$

$$= \theta^{(n+1)-1} (1-\theta)^{(\Sigma x_i + 1)-1} \, I(0 \le \theta \le 1)$$

$$\propto \frac{\Gamma(n + \sum_{i=1}^{n} x_i + 2)}{\Gamma(n+1)\,\Gamma(\sum_{i=1}^{n} x_i + 1)} \theta^{(n+1)-1} (1-\theta)^{[\Sigma x_i + 1]-1} \, I(0 \le \theta \le 1)$$

$$\text{Beta}\left(\alpha = n+1, \beta = \sum_{i=1}^{n} x_i + 1\right)$$

2. (30 Points) A random sample of size $n = 150$ yields $\sum_{i=1}^{n} = 163$. Estimate θ with the posterior expected value. The answer is a number.

$$E(\theta \mid \underset{\sim}{x}) = \frac{\alpha}{\alpha + \beta} = \frac{n+1}{n + \Sigma x_i + 2} = \frac{151}{150 + 163 + 2}$$

$$= \frac{151}{315} \doteq 0.479$$