Continuous Random Variables[1]

STA 256: Fall 2018

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Continuous Random Variables: The idea

Probability is area under a curve

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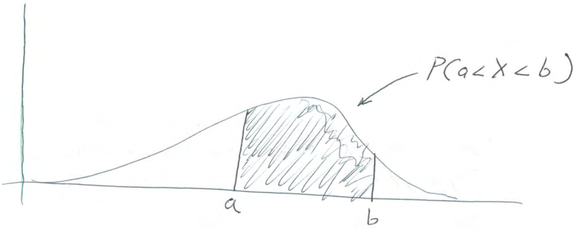

The Probability Density Function

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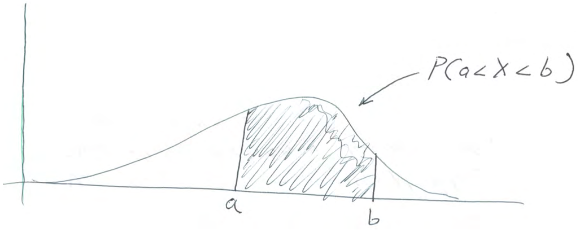

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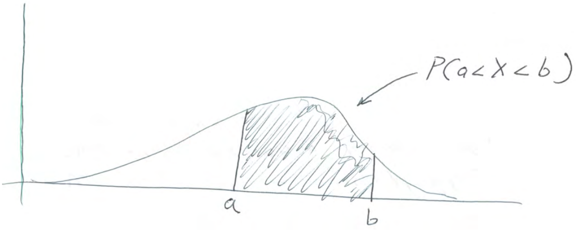

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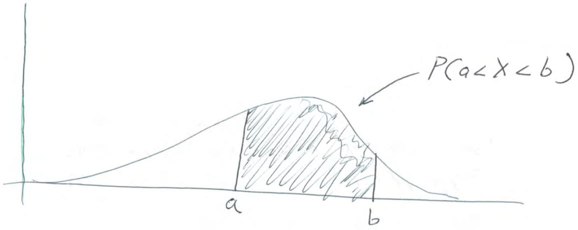

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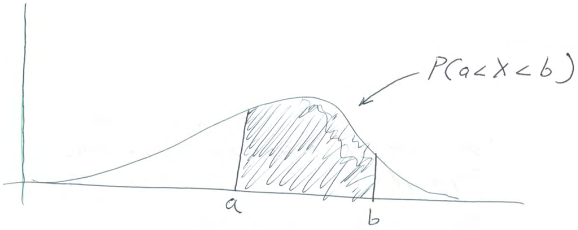

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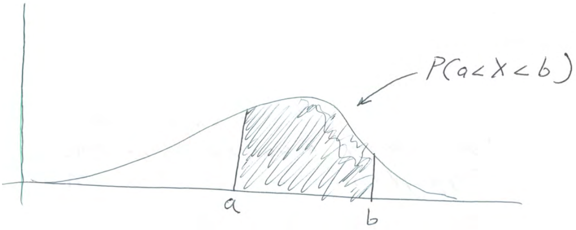

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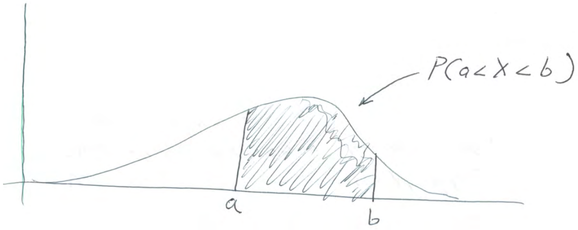

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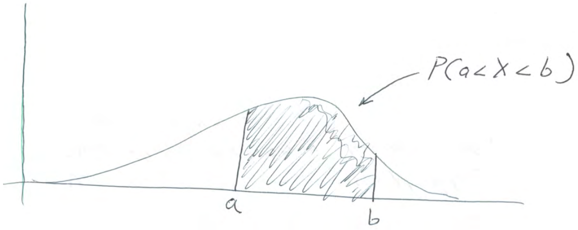

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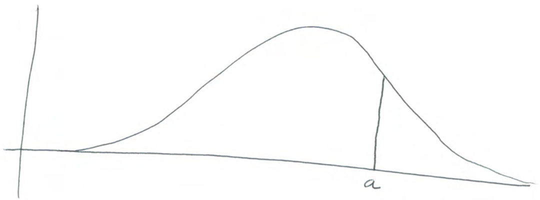
$f(x)$ is called the *density function* of X. Properties are

- $f(x) \geq 0$
- $f(x)$ is piecewise continuous.
- $\int_{-\infty}^{\infty} f(x)\, dx = 1$

The probability of any individual value of X is zero

$$P(X = a) = 0$$



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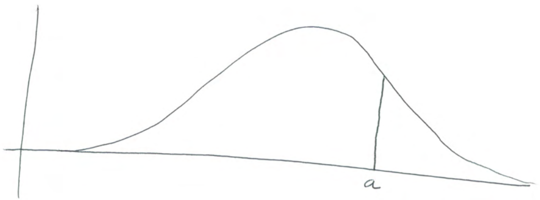
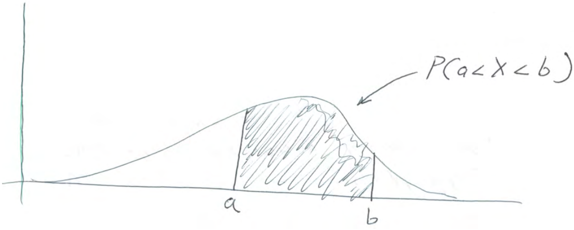


So
$P(< X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b)$.



$$P(a < X < b) = F(b) - F(a)$$

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By the Fundamental Theorem of Calculus.

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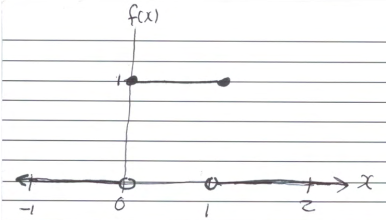

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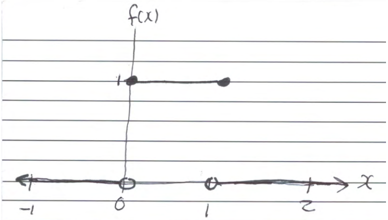

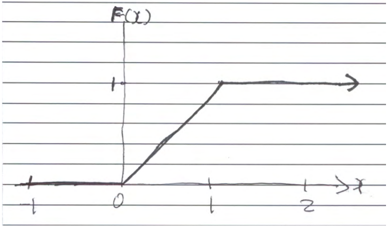

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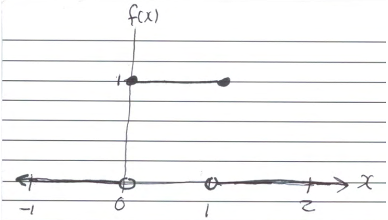

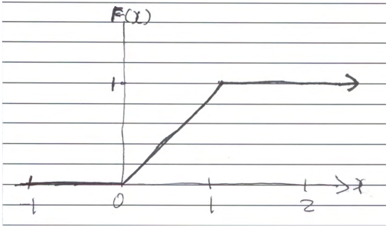

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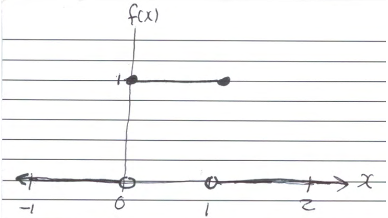
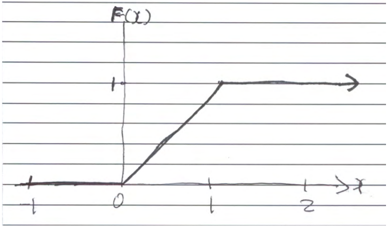
More comments





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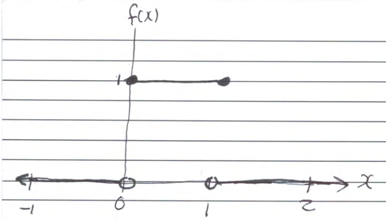

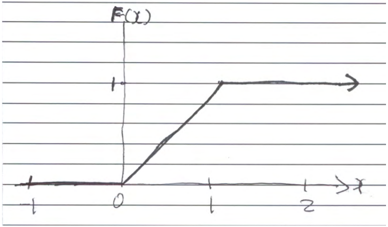

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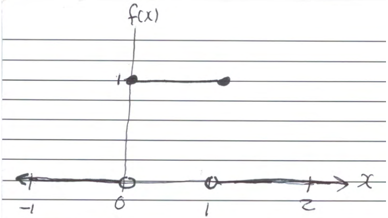

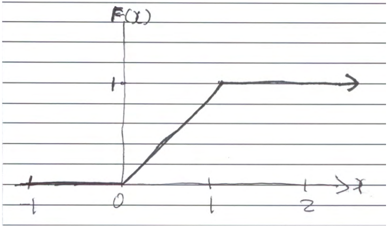

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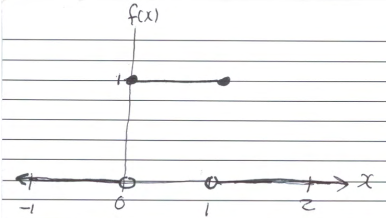

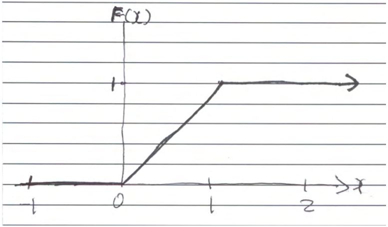

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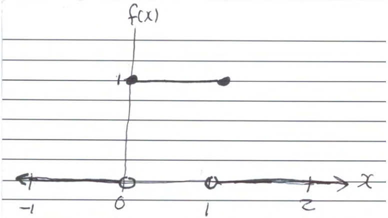

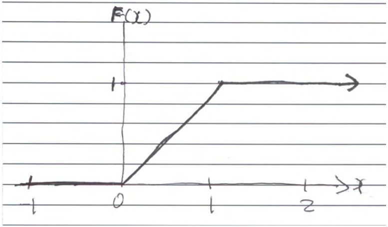

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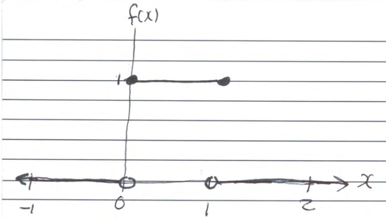

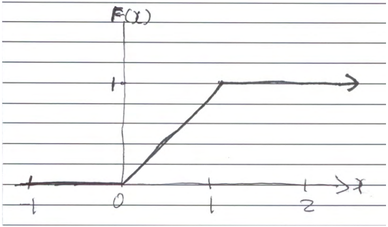

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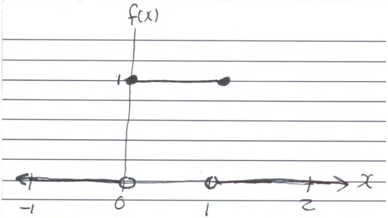

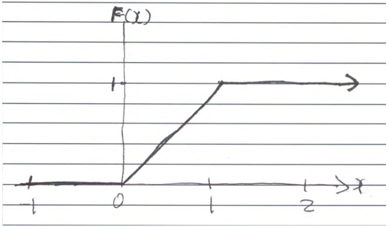

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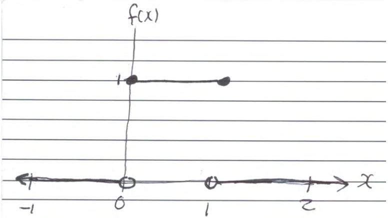

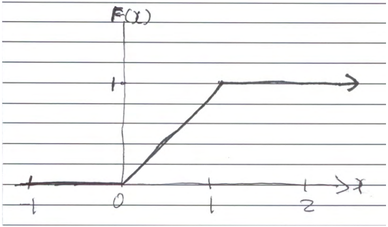

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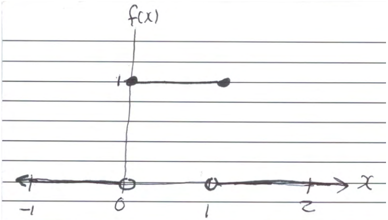

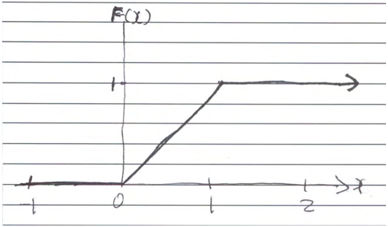

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$f(x)$ is not a probability

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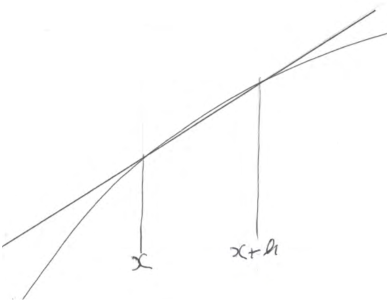

Another way to write $f(x)$

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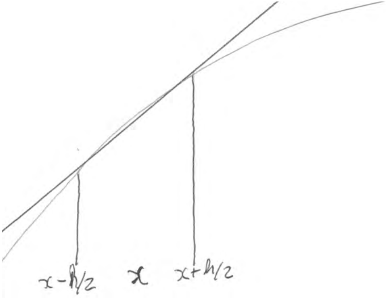
$$f(x) = \lim_{h\to 0} \frac{F(x+\frac{h}{2}) - F(x-\frac{h}{2})}{h}$$



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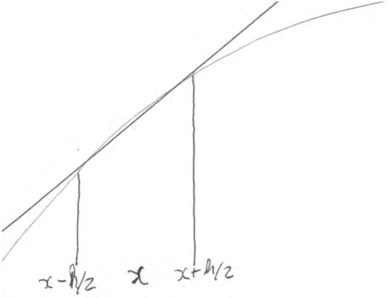

Limiting slope is the same if it exists.

Interpretation

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- $F(x + \frac{h}{2}) - F(x - \frac{h}{2}) = P(x - \frac{h}{2} < X < x + \frac{h}{2})$
- So $f(x)$ is roughly proportional to the probability that X is in a tiny interval surrounding x.

Example

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Common questions:

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Common questions:

- Prove it's a density.
- Find $F(x)$.

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$$\int_{-\infty}^{\infty} f(x)\, dx = \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{1} f(x)\, dx + \int_{1}^{\infty} f(x)\, dx$$

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The derivation does not need to be this detailed, but the final result has to be complete. More examples will be given.

Common Continuous Distributions

- Uniform
- Exponential
- Gamma
- Normal
- Beta

The Uniform Distribution: $X \sim \text{Uniform}(a, b)$

Parameters $a < b$

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{Otherwise} \end{cases}$$

The Uniform Distribution: $X \sim \text{Uniform}(a, b)$

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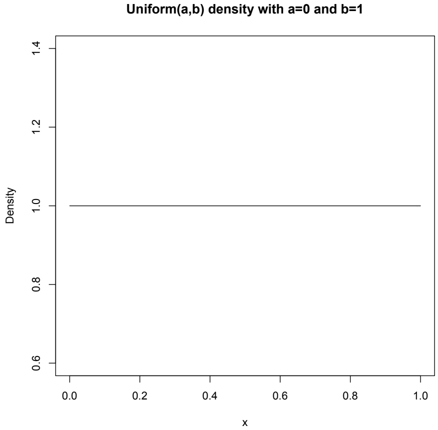

The Exponential Distribution: $X \sim \text{Exponential}(\lambda)$

Parameter $\lambda > 0$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

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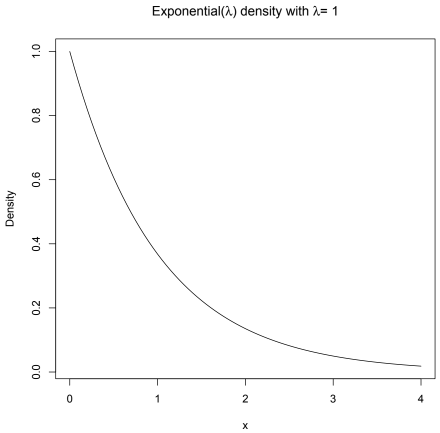

The Gamma Distribution: $X \sim \text{Gamma}(\alpha, \lambda)$

Parameters $\alpha > 0$ and $\lambda > 0$

$$f(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} \, x^{\alpha - 1} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

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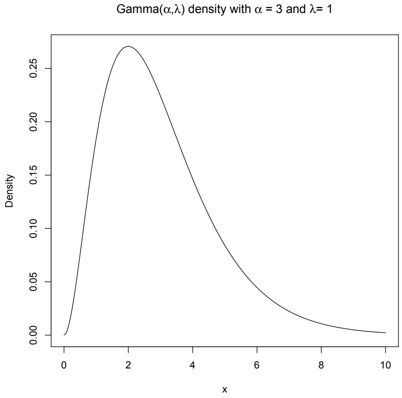

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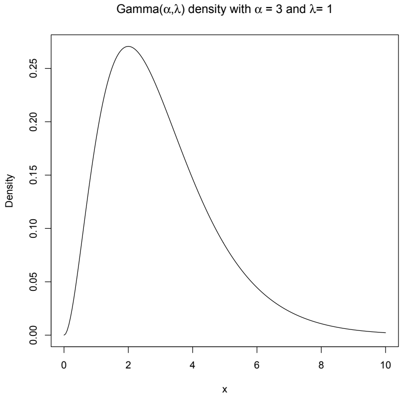

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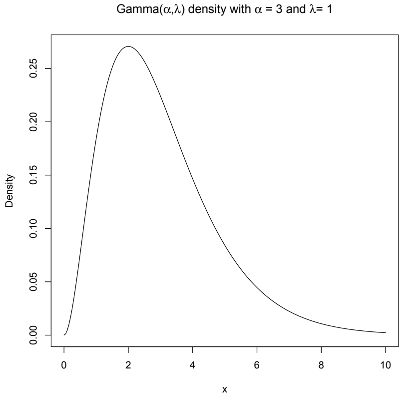

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Integration by parts shows $\Gamma(\alpha+1) = \alpha\, \Gamma(\alpha)$.

The Normal Distribution: $X \sim \mathrm{N}(\mu, \sigma)$

Parameters $\mu \in \mathbb{R}$ and $\sigma > 0$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

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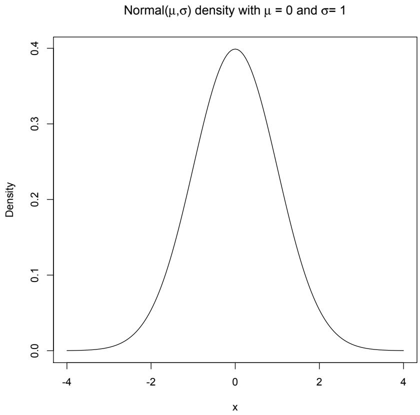

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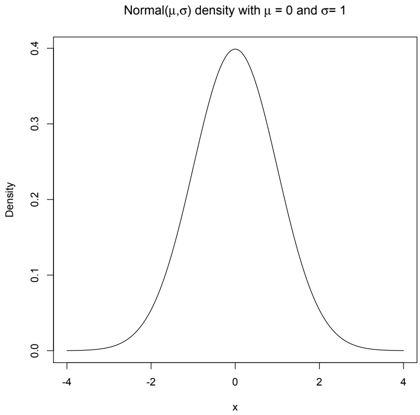

The normal distribution is also called the Gaussian, or the “bell curve.” if $\mu = 0$ and $\sigma = 1$, we write $X \sim \mathrm{N}(0,1)$ and call it the

The Beta Distribution: $X \sim \text{Beta}(\alpha, \beta)$

Parameters $\alpha > 0$ and $\beta > 0$

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{Otherwise} \end{cases}$$

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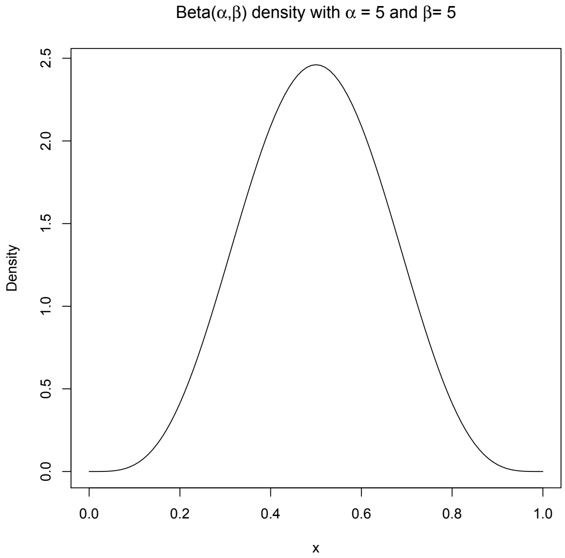
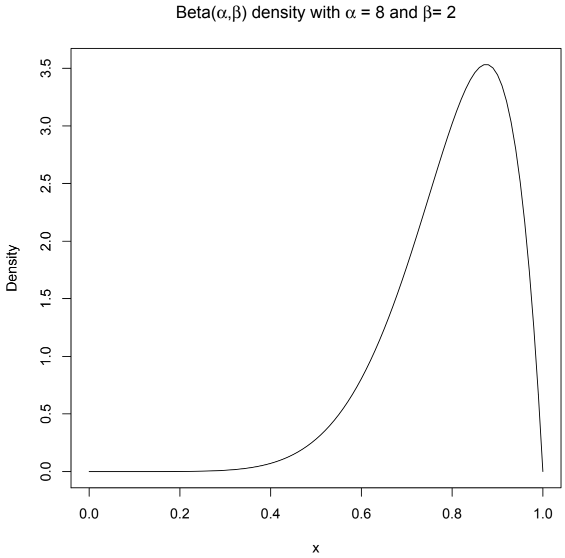
The beta density can assume a variety of shapes, depending on the parameters α and β.

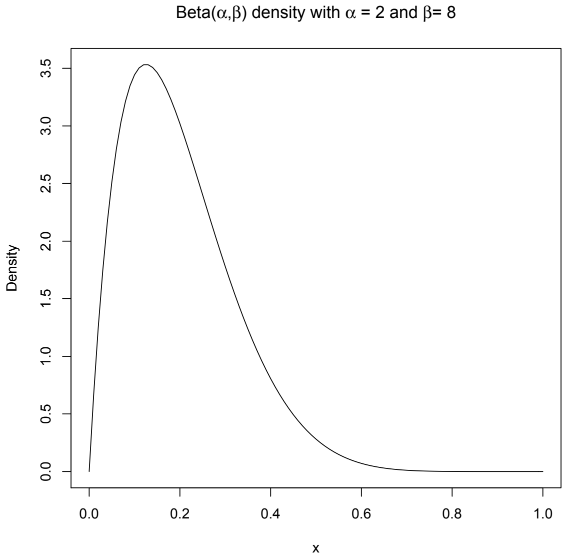
Beta density with $\alpha = 5$ and $\beta = 5$



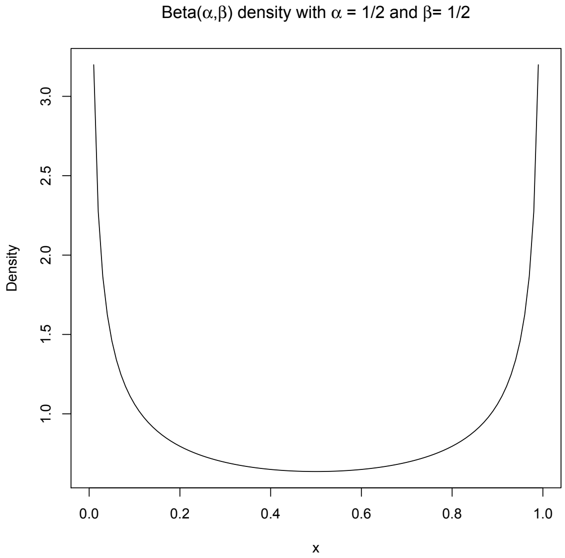
Beta density with $\alpha = 8$ and $\beta = 2$



Beta density with $\alpha = 2$ and $\beta = 8$



Beta density with $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$



Functions of a Random Variable

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- Y is litres per 100 kilometers for the same population of cars.
- You can make probability statements about X, but you need to make probability statements about Y

General approach to finding the density of $Y = g(X)$

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- We need an example.

Example

Let $X \sim \text{Gamma}(\alpha, \lambda)$

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- $x \geq 0 \Leftrightarrow y = 2x \geq 0$.

Example

Let $X \sim \text{Gamma}(\alpha, \lambda)$ and $Y = 2X$. Find the density of Y.

$$f_x(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

First, where will $f_y(y)$ be non-zero?

- $f_x(x) > 0$ for $x \geq 0$.
- $x \geq 0 \Leftrightarrow y = 2x \geq 0$.
- So, for $y \geq 0$, ...

Derive the functional part of $f_y(y)$

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f_y(y) &= \frac{d}{dy} F_y(y) \\
&= \frac{d}{dy} P(Y \leq y)
\end{aligned}
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Derive the functional part of $f_y(y)$

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$$
\begin{aligned}
&= f_x\left(\frac{1}{2}y\right) \cdot \frac{d}{dy}\frac{1}{2}y \\
&= \frac{1}{2} \cdot \frac{\lambda^\alpha}{\Gamma(\alpha)} \exp\left\{-\lambda\frac{1}{2}y\right\} \left(\frac{1}{2}y\right)^{\alpha-1}
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Compare gamma density: $f_x(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1}$ for $x \geq 0$.

Derive the functional part of $f_y(y)$

$X \sim \text{Gamma}(\alpha, \lambda)$ and $Y = 2X$

$$\begin{aligned} f_y(y) &= \frac{d}{dy} F_y(y) \\ &= \frac{d}{dy} P(Y \le y) \\ &= \frac{d}{dy} P(2X \le y) \\ &= \frac{d}{dy} P\left(X \le \frac{1}{2}y\right) \\ &= \frac{d}{dy} F_x\left(\frac{1}{2}y\right) \end{aligned}$$

$$\begin{aligned} &= f_x\left(\frac{1}{2}y\right) \cdot \frac{d}{dy}\frac{1}{2}y \\ &= \frac{1}{2} \cdot \frac{\lambda^\alpha}{\Gamma(\alpha)} \exp\left\{-\lambda \frac{1}{2}y\right\} \left(\frac{1}{2}y\right)^{\alpha-1} \\ &= \frac{(\lambda/2)^\alpha}{\Gamma(\alpha)} \exp\left\{-\frac{\lambda}{2}y\right\} y^{\alpha-1} \end{aligned}$$

Compare gamma density: $f_x(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1}$ for $x \ge 0$. So $Y \sim \text{Gamma}(\alpha, \lambda/2)$.

Give the density of Y

Don't forget to specify where $f_y(y) > 0$

$$f_y(y) = \begin{cases} \frac{(\lambda/2)^\alpha}{\Gamma(\alpha)} \exp\left\{-\frac{\lambda}{2} y\right\} y^{\alpha-1}, & \text{for } y \geq 0 \\ 0 & \text{for } y < 0 \end{cases}$$

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