Ch 9 Comparing Two means

Two main Settings

- Comparing two pre-existing groups

  Ex < Ever made a claim / Never made a claim } compare mean amount of claim

  Ex < Male / Female } compare mean GPA

  Ex < Receive Head Start / Not } compare ed test scores

(Note: Pre-existing groups are probably not comparable on something relevant)

- Experimental vs Control. Random assignment

  Ex < Drug / Placebo

  Ex < Ed / Program

Big distinction between

- Independent Groups
- Paired comparison

First, Independent Groups

Two Populations

$$\begin{cases} \mu_1, \sigma_1^2 & p_1 \\ & \text{or} \\ \mu_2, \sigma_2^2 & p_2 \end{cases}$$

Want to test $H_0: \mu_1 - \mu_2 = D_0$

usually $D_0 = 0$ and we test

$$\mathbf{H_0: \mu_1 = \mu_2} \quad \text{vs} \quad H_a: \mu_1 > \mu_2$$
$$H_a: \mu_1 < \mu_2$$
$$H_a: \mu_1 \neq \mu_2$$

or $\quad H_0: p_1 - p_2 = D_0$

usually $D_0 = 0$ and we test

$$\mathbf{H_0: p_1 = p_2} \quad \text{vs} \quad H_a: p_1 > p_2$$
$$H_a: p_1 < p_2$$
$$H_a: p_1 \neq p_2$$

IGNORE Tests, confidence intervals for the variances

IGNORE Confidence intervals for $\mu_1 - \mu_2$ & $p_1 - p_2$

Tests for Two Independent Groups

- Large-Sample Z-test for Means   P. 433

  $n_1 \geq 30$, $n_2 \geq 30$

  $\sigma_1^2 \neq \sigma_2^2$ OKAY

  Non-normal Distributions OKAY

- Two-Sample T-test   P. 437

  n_1, n_2 can be small

  $\sigma_1^2 = \sigma_2^2$ when H_0 is true

  Normal Distributions

- Large-Sample Z-test for 2 proportions

  P. 471

  $P \pm 3\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ Does not include 0 o

  where $P_1 = P_2 = P$

Large-Sample z-test for Means

A random sample of 214 American managers and another random sample of 109 Japanese managers took an attitude scale intended to measure motivation for upward mobility. Here are the results.

	American	Japanese
Mean	76.8	79.71
Standard Deviation	11.06	6.43
n	214	109

We wish to know whether American & Japanese managers differ in their average motivation level. Use $\alpha = 0.05$.

$$H_0: \mu_1 = \mu_2 \quad \text{vs} \quad H_a: \mu_1 \neq \mu_2$$

(e)

	American	Japanese
$\bar{X}$	76.8	79.71
s	11.06	6.43
n	214	109

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{76.8 - 79.71}{\sqrt{\frac{11.06^2}{214} + \frac{6.43^2}{109}}}$$

$$= \frac{-2.91}{\sqrt{.5716 + .3793}} = -2.98$$

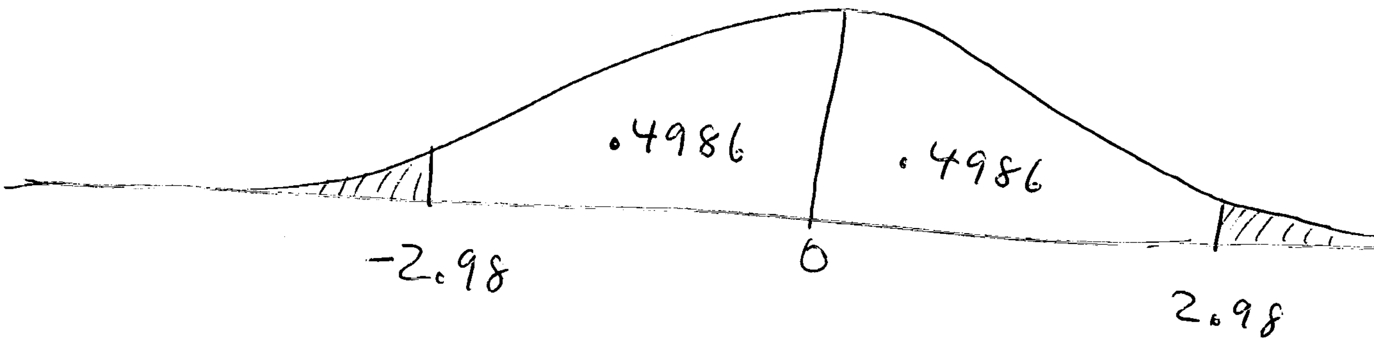
Rejection Region: $|z| > 1.96$

Reject H_0? Yes

Conclude Japanese managers get higher motivation scores on average

Are results Statistically Significant? Yes.

p- value



$P = 2(.5 - .4986) = .0028$

Reject H_0 at $\alpha = .01$ Yes

$\alpha = .001$ No

Two-sample t-test

- n_1, n_2 can be small (or large)
- $\sigma_1^2 = \sigma_2^2$ when H_0 is true
  (plausible for random assignment to experimental vs control group)
- Normal distributions within groups

Assume equal variances. Estimate the common value $\sigma^2 = \sigma_1^2 = \sigma_2^2$ with

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad df = n_1 + n_2 - 2$$

EX Calcium uptake in fast-twitch vs slow-twitch muscles in crayfish (problem 9.132). Assume normal. Samples are independent.

Fast-Twitch Muscle	Slow-twitch muscle
$n_1 = 12$	$n_2 = 12$
$\bar{X}_1 = .57$	$\bar{X}_2 = .37$
$S_1 = .104$	$S_2 = .035$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{11(.104)^2 + 11(.035)^2}{22}$$

$$= \frac{.132451}{22} = .00602$$

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.57 - .37}{\sqrt{.00602\left(\frac{1}{12} + \frac{1}{12}\right)}}$$

$$= \frac{.2}{.031675} = 6.31$$

Critical value for 2-tailed test is 3.792 at $\alpha = .001$. Definitely conclude there is an effect. Say "There was significantly more calcium uptake for fast-twitch muscles ($t = 6.31$, $df = 22$, $p < .001$ two-tailed)"

Possible Trick questions

* Forget to mention normal — Stop. Impossible with our technology. Try a non parametric test.

* Want to determine whether calcium uptake is greater for slow-twitch muscles. $H_0: \mu_1 = \mu_2$ vs $H_a: \mu_1 < \mu_2$
Critical region $t < -1.717$ at $\alpha = 0.05$
Don't reject H_0, conclude nothing, insufficient evidence, SCIENTIFICALLY FOOLISH.

Large - Sample Z-test for two proportions

$H_0: p_1 = p_2$

CLT yields large-sample normality for sampling distribution of $\hat{p}_1 - \hat{p}_2$. Variance of sampling distribution depends on the common value of $p = p_1 = p_2$ (Assuming $H_0: p_1 = p_2$ true). Estimate with

$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$

Check: Does $\hat{p} \pm 3\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ Include 0 or 1?

If not,

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \text{ where } \hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$

Ex Of a random sample of 150 Special Needs students in the Toronto District School board, 19 were in regular classes, and the rest were in Special Education classes. Of a random sample of 200 students in the Toronto Separate School Board, 48 were in regular classes. Test for difference between proportions of Special Needs students in the two school boards, using $\alpha = 0.01$.

$\hat{p}_1 = \frac{19}{150} = .1267$, $\hat{p}_2 = \frac{48}{200} = .24$

$\hat{p} = \frac{19+48}{350} = .1914$, $\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = .0.425$

$\hat{q} = .8086$

AND $.1914 \pm 3(.0425) = .1914 \pm .1275$ does not include 0 or 1, ok

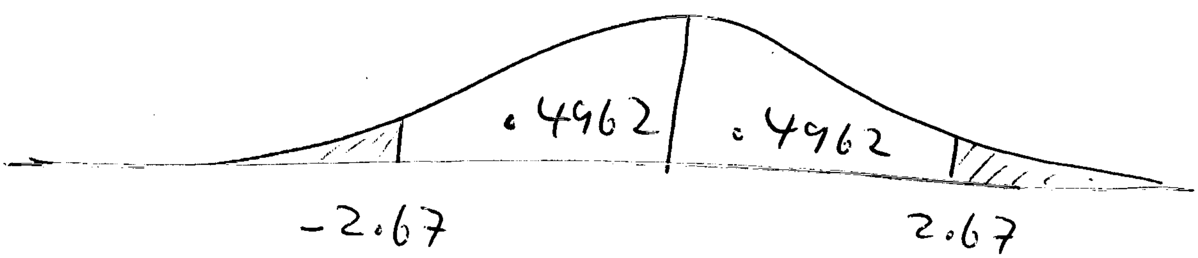
$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{.1267 - .24}{.0425} = -2.67$$

e

$Z = -2.67$

Rejection Region: $|z| > 2.567$

Reject H_0! Conclude that special needs students are more likely to be in regular classes in the separate Board.



$p = 1 - 2 * .4962 = .0076$

At $\alpha = .001$, H_0 would not be rejected. We would not say the proportions of special needs students in regular classes was the SAME. We would say there is insufficient evidence to conclude that the proportions of students in regular classes were different.

PAIRED DIFFERENCE Experiments

SAME INDIVIDUAL (OR UNIT) measured twice under different conditions.

Examples

- Pretest Treatment Post-test
- Psychophysics: lots of trials under condition A, lots under B, random order. Average response to A, Av response to B.
- Longevity of Husbands & Wives
- Litter-mates (Drug study)

IT'S A MISTAKE to USE INDEPENDENT GROUPS FORMULAS ON DATA LIKE THESE, because then you are pretending you have $2n$ independent pieces of information. You will think you have more precision than you really do.

Compute differences for each individual or unit $n = \#$ of differences

Difference between means equals the mean difference (Sample and Population)

Test hypotheses about mean differences using familiar one-sample methods

Section 9.3

$$Z = \frac{\bar{X}_D - D_0}{S_D / \sqrt{n_D}}$$

$$t = \frac{\bar{X}_D - D_0}{S_D / \sqrt{n_D}}$$

Ch. 8

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

$$t = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

Assumptions (for example normality for the t-test when $n < 30$) apply to Differences