Power via Non-centrality Parameters

For large sample chi-square tests

Wald Tests

- Test Statistic(s)

$$W_1 = (\mathbf{C}\widehat{\boldsymbol{\theta}} - \mathbf{h})'(\mathbf{C}\mathbf{H}(\widehat{\theta})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\boldsymbol{\theta}} - \mathbf{h})$$

$$W_2 = n\,(\mathbf{C}\widehat{\boldsymbol{\theta}} - \mathbf{h})'(\mathbf{C}\mathbf{I}(\widehat{\theta})^{-1}\mathbf{C}')^{-1}(\mathbf{C}\widehat{\boldsymbol{\theta}} - \mathbf{h})$$

  - W_1 uses the Hessian – the observed Fisher information at theta-hat
  - W_2 uses the original Fisher information in a single observation. Need to take the expected value by hand.

- Non-centrality parameter

$$\phi = n\,(\mathbf{C}\boldsymbol{\theta} - \mathbf{h})'(\mathbf{C}\mathbf{I}(\theta)^{-1}\mathbf{C}')^{-1}(\mathbf{C}\boldsymbol{\theta} - \mathbf{h})$$

Special formulas for multinomial models

- Pearson Chisquare test
- Likelihood ratio test

- Let π_i denote the cell probabilities.
- Let f_o denote the observed frequencies.
- Let $f_e = n\,\widehat{\pi}_{0,i}$ denote the estimated expected frequencies under H_0
- Let π(M) denote the value to which pi-hat converges under the null hypothesis model.

Pearson Chisquare test

$$X^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$\phi = n \sum_i \frac{(\pi_i - \pi_i(M))^2}{\pi_i(M)}$$

Likelihood Ratio Tests

$$G = 2 \sum f_o \log \left(\frac{f_o}{f_e} \right)$$

$$\phi = 2n \sum_i \pi_i \log \left(\frac{\pi_i}{\pi_i(M)} \right)$$

General Likelihood Ratio Tests

- $\theta = (\theta_1, \ldots, \theta_r, \theta_{r+1}, \ldots, \theta_{r+s})$
- H_0: $\theta_1 = h_1, \ldots, \theta_r = h_r$ or write it
- H_0: $\boldsymbol{\theta}_r = \mathbf{h}$
- If H_0 is not of this form, re-parameterize
  - First *r* new parameters are functions set to $h_1, \ldots, h_r$ by H_0
  - Remaining *s* new parameters make the re-parameterization one-to-one.

Likelihood Ratio Tests

$$X_1, \ldots, X_n \overset{i.i.d.}{\sim} F_\theta,\ \theta \in \Theta$$
$$\Theta_0 = \{\theta \in \Theta : \theta_1 = h_1, \ldots, \theta_r = h_r\}$$
$$H_0 : \theta \in \Theta_0 \text{ v.s. } H_A : \theta \in \Theta \cap \Theta_0^c,$$

$$G = -2 \log \left(\frac{\sup_{\theta \in \Theta_0} L(\theta)}{\sup_{\theta \in \Theta} L(\theta)} \right)$$

$$\phi = n\,(\boldsymbol{\theta}_r - \mathbf{h})' \mathbf{I}_r(\theta)(\boldsymbol{\theta}_r - \mathbf{h})$$

$\mathbf{I}_r(\theta)$ is the upper left *rxr* part of the Fisher information matrix for a single observation. See ATS p. 869.

Compare

$$\phi = 2n \sum_i \pi_i \log \left(\frac{\pi_i}{\pi_i(M)} \right)$$

$$\phi = n\,(\boldsymbol{\theta}_r - \mathbf{h})' \mathbf{I}_r(\theta)(\boldsymbol{\theta}_r - \mathbf{h})$$