Exploratory and Attempted Conformatory Factor Analysis for a Non-identified Two-factor Model

```
/* fac2.sas */
options linesize=79 pagesize=200noovp formdlim='_';
title 'Confirmatory and Exploratory factor analysis with simulated data';
title2 'Non-identified 2-factor model';

data latent1; /* Variables load only on one factor, but we don't know it. */
     n = 500; iseed = 51493;

     /* True factor loadings (All communalities = .25) */
     L11 = .5; L12 =  0;
     L21 = .5; L22 =  0;
     L31 = .5; L32 =  0;
     L41 = .5; L42 =  0;
     L51 =  0; L52 = .5;
     L61 =  0; L62 = .5;
     L71 =  0; L72 = .5;
     L81 =  0; L82 = .5;

     /* Variances of error terms will all equal 0.75*/
     v1 = 1 - L11**2 - L12**2;
     v2 = 1 - L21**2 - L22**2;
     v3 = 1 - L31**2 - L32**2;
     v4 = 1 - L41**2 - L42**2;
     v5 = 1 - L51**2 - L52**2;
     v6 = 1 - L61**2 - L62**2;
     v7 = 1 - L71**2 - L72**2;
     v8 = 1 - L81**2 - L82**2;

     do i=1 to n;
          /* Factors are independent standard normal */
          F1 = rannor(iseed); F2 = rannor(iseed);
          /* Observed variables all have variance one */
          X1 = L11*F1 + L12*F2 + sqrt(v1)*rannor(iseed);
          X2 = L21*F1 + L22*F2 + sqrt(v2)*rannor(iseed);
          X3 = L31*F1 + L32*F2 + sqrt(v3)*rannor(iseed);
          X4 = L41*F1 + L42*F2 + sqrt(v4)*rannor(iseed);
          X5 = L51*F1 + L52*F2 + sqrt(v5)*rannor(iseed);
          X6 = L61*F1 + L62*F2 + sqrt(v6)*rannor(iseed);
          X7 = L71*F1 + L72*F2 + sqrt(v7)*rannor(iseed);
          X8 = L81*F1 + L82*F2 + sqrt(v8)*rannor(iseed);
          output; /* Create a case */
     end;
     keep x1-x8;
```

```
proc calis pcorr; /* Print correlation matrix */
     title3 'Try to fit non-identified 2-factor model with proc calis';
     var X1-X8;          /* Manafest vars are in the data set */
     lineqs              /* Simultaneous equations, separated by commas */
          X1 = lambda11 F1 + lambda12 F2 + delta1,
          X2 = lambda21 F1 + lambda22 F2 + delta2,
          X3 = lambda31 F1 + lambda32 F2 + delta3,
          X4 = lambda41 F1 + lambda42 F2 + delta4,
          X5 = lambda51 F1 + lambda52 F2 + delta5,
          X6 = lambda61 F1 + lambda62 F2 + delta6,
          X7 = lambda71 F1 + lambda72 F2 + delta7,
          X8 = lambda81 F1 + lambda82 F2 + delta8;

     std                      /* Variances (not standard deviations) */
          F1 = 1 , F2 = 1,
          delta1-delta8 = theta1-theta8;
     bounds 0.0 < theta1-theta8;    /* Variances are positive */

proc factor method=ML rotate=varimax;
     title3 'Estimate Loadings with exploratory factor analysis';
     var X1 - X8;

/* Try starting proc calis at the rotated solution */

proc calis;
     title3 'Start at rotated solution: Objective Function = 0.046133184?';
     var X1-X8;          /* Manafest vars are in the data set */
     lineqs              /* Simultaneous equations, separated by commas */
          X1 = lambda11 (0.44623) F1 + lambda12 (-0.00138) F2 + delta1,
          X2 = lambda21 (0.61703) F1 + lambda22 (-0.10112) F2 + delta2,
          X3 = lambda31 (0.49899) F1 + lambda32 (-0.05083) F2 + delta3,
          X4 = lambda41 (0.47995) F1 + lambda42 (0.12903) F2 + delta4,
          X5 = lambda51 (0.03523) F1 + lambda52 (0.49182) F2 + delta5,
          X6 = lambda61 (-0.04097) F1 + lambda62 (0.47610) F2 + delta6,
          X7 = lambda71 (0.04314) F1 + lambda72 (0.43286) F2 + delta7,
          X8 = lambda81 (-0.07363) F1 + lambda82 (0.58452) F2 + delta8;

     std                      /* Variances (not standard deviations) */
          F1 = 1 , F2 = 1,
          delta1-delta8 = theta1-theta8;
     bounds 0.0 < theta1-theta8;    /* Variances are positive */
```

```
data latent2; /* Variables load on both factors */
     n = 500; iseed = 51493;

     /* True factor loadings (All communalities = .5^2 + .8^2 = 0.89) */
     L11 = .5; L12 =  -.8;
     L21 = .5; L22 =  -.8;
     L31 = .5; L32 =  -.8;
     L41 = .5; L42 =  -.8;
     L51 = .8; L52 = .5;
     L61 = .8; L62 = .5;
     L71 = .8; L72 = .5;
     L81 = .8; L82 = .5;

     /* Variances of error terms all equal 0.11 */
     v1 = 1 - L11**2 - L12**2;
     v2 = 1 - L21**2 - L22**2;
     v3 = 1 - L31**2 - L32**2;
     v4 = 1 - L41**2 - L42**2;
     v5 = 1 - L51**2 - L52**2;
     v6 = 1 - L61**2 - L62**2;
     v7 = 1 - L71**2 - L72**2;
     v8 = 1 - L81**2 - L82**2;

     do i=1 to n;
          /* Factors are independent standard normal */
          F1 = rannor(iseed); F2 = rannor(iseed);
          /* Observed variables all have variance one */
          X1 = L11*F1 + L12*F2 + sqrt(v1)*rannor(iseed);
          X2 = L21*F1 + L22*F2 + sqrt(v2)*rannor(iseed);
          X3 = L31*F1 + L32*F2 + sqrt(v3)*rannor(iseed);
          X4 = L41*F1 + L42*F2 + sqrt(v4)*rannor(iseed);
          X5 = L51*F1 + L52*F2 + sqrt(v5)*rannor(iseed);
          X6 = L61*F1 + L62*F2 + sqrt(v6)*rannor(iseed);
          X7 = L71*F1 + L72*F2 + sqrt(v7)*rannor(iseed);
          X8 = L81*F1 + L82*F2 + sqrt(v8)*rannor(iseed);
          output; /* Create a case */
     end;

proc factor method=ML rotate=varimax;
title3 'Expl. factor analysis when all vars really load on both factors';
     var X1 - X8;
```

The CALIS Procedure
Covariance Structure Analysis: Pattern and Initial Values

LINEQS Model Statement

		Matrix	Rows	Columns	------Matrix Type-------	
Term 1	1	_SEL_	8	18	SELECTION	
	2	_BETA_	18	18	EQSBETA	IMINUSINV
	3	_GAMMA_	18	10	EQSGAMMA	
	4	_PHI_	10	10	SYMMETRIC	

The 8 Endogenous Variables

Manifest	X1	X2	X3	X4	X5	X6	X7	X8
Latent								

The 10 Exogenous Variables

Manifest								
Latent	F1	F2						
Error	delta1	delta2	delta3	delta4	delta5	delta6	delta7	delta8

Correlations

	X1	X2	X3	X4	X5	X6	X7	X8
X1	1.0000	0.2564	0.2431	0.2183	0.0400	-0.0195	0.0151	-0.0521
X2	0.2564	1.0000	0.3190	0.2945	-0.0214	-0.0184	-0.0417	-0.1343
X3	0.2431	0.3190	1.0000	0.2107	-0.0055	-0.1010	-0.0125	-0.0164
X4	0.2183	0.2945	0.2107	1.0000	0.0438	0.0173	0.1285	0.0530
X5	0.0400	-0.0214	-0.0055	0.0438	1.0000	0.2790	0.2253	0.2549
X6	-0.0195	-0.0184	-0.1010	0.0173	0.2790	1.0000	0.1468	0.2926
X7	0.0151	-0.0417	-0.0125	0.1285	0.2253	0.1468	1.0000	0.2662
X8	-0.0521	-0.1343	-0.0164	0.0530	0.2549	0.2926	0.2662	1.0000

Determinant 0.493401 Ln -0.706433

NOTE: Some initial estimates computed by approximative factor analysis.

Skipping listing of starting values …

Levenberg-Marquardt Optimization

Scaling Update of More (1978)

Parameter Estimates	24
Functions (Observations)	36
Lower Bounds	8
Upper Bounds	0

Optimization Start

Active Constraints	0	Objective Function	0.0920853206
Max Abs Gradient Element	0.1649778516	Radius	1

Iter	Rest arts	Func Calls	Act Con	Objective Function	Obj Fun Change	Max Abs Gradient Element	Lambda	Actual Over Pred Change
1*	0	2	0	0.04786	0.0442	0.0469	111E-16	0.866
2	0	3	0	0.04618	0.00168	0.00441	0	0.975
3*	0	4	0	0.04614	0.000045	0.00173	111E-16	0.900
4*	0	5	0	0.04613	3.403E-6	0.000272	111E-16	0.899
5	0	6	0	0.04613	2.489E-7	0.000173	0	0.826
6*	0	7	0	0.04613	4.553E-8	0.000022	111E-16	0.949
7*	0	8	0	0.04613	1.947E-9	9.409E-6	111E-16	0.911

Optimization Results

Iterations	7	Function Calls	9
Jacobian Calls	8	Active Constraints	0
Objective Function	0.0461331842	Max Abs Gradient Element	9.4087288E-6
Lambda	1.110223E-14	Actual Over Pred Change	0.9105070821
Radius	0.2773814698		

ABSGCONV convergence criterion satisfied.

NOTE: Moore-Penrose inverse is used in covariance matrix.

NOTE: Covariance matrix for the estimates is not full rank.

NOTE: The variance of some parameter estimates is zero or some parameter estimates are linearly related to other parameter estimates as shown in the following equations:

```
 lambda21   =   1.101807E-16   -       0.793600   *   lambda11   -
                              0.440518   *   lambda12   -       0.785901
                             *   lambda22   -       0.839060   *   lambda31
                             -       0.580388   *   lambda32   -
                              0.981720   *   lambda41   -       0.241288
                             *   lambda42   -       0.545552   *   lambda51
                             +       0.841570   *   lambda52   -
                              0.394371   *   lambda61   +       0.888376
                             *   lambda62   -       0.501799   *   lambda71
                             +       0.728809   *   lambda72   -
                              0.442683   *   lambda81   +       1.113665
                             *   lambda82
```

Predicted Model Matrix

	X1	X2	X3	X4	X5	X6	X7	X8
X1	1.0000	0.2755	0.2227	0.2140	0.0150	-0.0189	0.0187	-0.0337
X2	0.2755	1.0000	0.3130	0.2831	-0.0280	-0.0734	-0.0172	-0.1045
X3	0.2227	0.3130	1.0000	0.2329	-0.0074	-0.0446	-0.0005	-0.0664
X4	0.2140	0.2831	0.2329	1.0000	0.0804	0.0418	0.0766	0.0401
X5	0.0150	-0.0280	-0.0074	0.0804	1.0000	0.2327	0.2144	0.2849
X6	-0.0189	-0.0734	-0.0446	0.0418	0.2327	1.0000	0.2043	0.2813
X7	0.0187	-0.0172	-0.0005	0.0766	0.2144	0.2043	1.0000	0.2499
X8	-0.0337	-0.1045	-0.0664	0.0401	0.2849	0.2813	0.2499	1.0000

Determinant 0.516696 Ln -0.660300

Non-identified 2-factor model
Try to fit non-identified 2-factor model with proc calis
22:17 Wednesday, April 5, 2006

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Fit Function	0.0461
Goodness of Fit Index (GFI)	0.9888
GFI Adjusted for Degrees of Freedom (AGFI)	0.9665
Root Mean Square Residual (RMR)	0.0261
Parsimonious GFI (Mulaik, 1989)	0.4238
Chi-Square	**23.0205**
Chi-Square DF	12
Pr > Chi-Square	0.0276
Independence Model Chi-Square	352.51
Independence Model Chi-Square DF	28
RMSEA Estimate	0.0429
RMSEA 90% Lower Confidence Limit	0.0139
RMSEA 90% Upper Confidence Limit	0.0692
ECVI Estimate	0.1441
ECVI 90% Lower Confidence Limit	0.1243
ECVI 90% Upper Confidence Limit	0.1797
Probability of Close Fit	0.6363
Bentler's Comparative Fit Index	0.9660
Normal Theory Reweighted LS Chi-Square	22.5125
Akaike's Information Criterion	-0.9795
Bozdogan's (1987) CAIC	-63.5548
Schwarz's Bayesian Criterion	-51.5548
McDonald's (1989) Centrality	0.9890
Bentler & Bonett's (1980) Non-normed Index	0.9208
Bentler & Bonett's (1980) NFI	0.9347
James, Mulaik, & Brett (1982) Parsimonious NFI	0.4006
Z-Test of Wilson & Hilferty (1931)	1.9184
Bollen (1986) Normed Index Rho1	0.8476
Bollen (1988) Non-normed Index Delta2	0.9676
Hoelter's (1983) Critical N	457

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Manifest Variable Equations with Estimates

X1	=	-0.2166*F1	+	0.3901*F2	+	1.0000 delta1
Std Err		0.0512 lambda11		0.0563 lambda12		
t Value		-4.2301		6.9247		
X2	=	-0.3864*F1	+	0.4916*F2	+	1.0000 delta2
Std Err		0.0495 lambda21		0.0556 lambda22		
t Value		-7.7996		8.8467		
X3	=	-0.2853*F1	+	0.4125*F2	+	1.0000 delta3
Std Err		0.0507 lambda31		0.0555 lambda32		
t Value		-5.6290		7.4326		
X4	=	-0.1186*F1	+	0.4826*F2	+	1.0000 delta4
Std Err		0.0483 lambda41		0.0581 lambda42		
t Value		-2.4568		8.3003		
X5	=	0.4137*F1	+	0.2682*F2	+	1.0000 delta5
Std Err		0.0556 lambda51		0.0522 lambda52		
t Value		7.4476		5.1393		
X6	=	0.4367*F1	+	0.1939*F2	+	1.0000 delta6
Std Err		0.0565 lambda61		0.0511 lambda62		
t Value		7.7329		3.7937		
X7	=	0.3583*F1	+	0.2467*F2	+	1.0000 delta7
Std Err		0.0554 lambda71		0.0534 lambda72		
t Value		6.4728		4.6164		
X8	=	0.5475*F1	+	0.2176*F2	+	1.0000 delta8
Std Err		0.0584 lambda81		0.0481 lambda82		
t Value		9.3775		4.5273		

Variances of Exogenous Variables

Variable	Parameter	Estimate	Standard Error	t Value
F1		1.00000		
F2		1.00000		
delta1	theta1	0.80088	0.06122	13.08
delta2	theta2	0.60903	0.06843	8.90
delta3	theta3	0.74843	0.06188	12.10
delta4	theta4	0.75299	0.06246	12.06
delta5	theta5	0.75689	0.06338	11.94
delta6	theta6	0.77167	0.06259	12.33
delta7	theta7	0.81077	0.06222	13.03
delta8	theta8	0.65288	0.06790	9.62

Non-identified 2-factor model

Estimate Loadings with exploratory factor analysis

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The FACTOR Procedure

Initial Factor Method: Maximum Likelihood

Prior Communality Estimates: SMC

X1	X2	X3	X4
0.11488172	0.19863374	0.15101564	0.14062067

X5	X6	X7	X8
0.13509327	0.14347560	0.11307973	0.17690305

Preliminary Eigenvalues: Total = 1.38529374 Average = 0.17316172

	Eigenvalue	Difference	Proportion	Cumulative
1	1.15802941	0.13336140	0.8359	0.8359
2	1.02466801	0.99242098	0.7397	1.5756
3	0.03224703	0.09301184	0.0233	1.5989
4	-.06076481	0.02558495	-0.0439	1.5550
5	-.08634975	0.05936803	-0.0623	1.4927
6	-.14571778	0.07765969	-0.1052	1.3875
7	-.22337747	0.09006343	-0.1612	1.2263
8	-.31344090		-0.2263	1.0000

2 factors will be retained by the PROPORTION criterion.

Iteration	Criterion	Ridge	Change	Communalities			
1	0.0462291	0.0000	0.1823	0.20296	0.38096	0.25435	0.24588
				0.25187	0.22740	0.18489	0.34224
2	0.0461358	0.0000	0.0093	0.19973	0.39030	0.25188	0.24633
				0.24371	0.22976	0.18923	0.34527
3	0.0461333	0.0000	0.0017	0.19917	0.39078	0.25168	0.24695
				0.24343	0.22842	0.18904	0.34696
4	0.0461332	0.0000	0.0003	0.19913	0.39095	0.25158	0.24699
				0.24313	0.22838	0.18923	0.34707

Convergence criterion satisfied.

Significance Tests Based on 500 Observations

Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	28	350.0376	<.0001
HA: At least one common factor			
H0: 2 Factors are sufficient	13	22.7975	0.0442
HA: More factors are needed			

Chi-Square without Bartlett's Correction	**23.020460**
Akaike's Information Criterion	-2.979540
Schwarz's Bayesian Criterion	-57.769445
Tucker and Lewis's Reliability Coefficient	0.934473

Squared Canonical Correlations

Factor1	Factor2
0.61304844	0.57492542

Eigenvalues of the Weighted Reduced Correlation Matrix: Total = 2.93683106 Average = 0.36710388

	Eigenvalue	Difference	Proportion	Cumulative
1	1.58430280	0.23177448	0.5395	0.5395
2	1.35252832	1.17146987	0.4605	1.0000
3	0.18105845	0.10067175	0.0617	1.0617
4	0.08038670	0.02220057	0.0274	1.0890
5	0.05818613	0.07874782	0.0198	1.1088
6	-.02056169	0.09392355	-0.0070	1.1018
7	-.11448523	0.07009920	-0.0390	1.0629
8	-.18458444		-0.0629	1.0000

Factor Pattern

	Factor1	Factor2
X1	0.38757	0.22115
X2	0.58536	0.21979
X3	0.45796	0.20458
X4	0.35183	0.35102
X5	-0.21452	0.44397
X6	-0.27276	0.39237
X7	-0.17829	0.39679
X8	-0.35509	0.47010

Variance Explained by Each Factor

Factor	Weighted	Unweighted
Factor1	1.58430280	1.10466437
Factor2	1.35252832	0.99177981

Final Communality Estimates and Variable Weights
Total Communality: Weighted = 2.936831 Unweighted = 2.096444

Variable	Communality	Weight
X1	0.19912149	1.24864087
X2	0.39095278	1.64189820
X3	0.25157813	1.33614951
X4	0.24699900	1.32799573
X5	0.24312507	1.32123679
X6	0.22835359	1.29597303
X7	0.18923091	1.23338999
X8	0.34708321	1.53154694

Non-identified 2-factor model
Estimate Loadings with exploratory factor analysis
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The FACTOR Procedure
Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2
1	0.86701	-0.49829
2	0.49829	0.86701

Rotated Factor Pattern

	Factor1	Factor2
X1	0.44623	-0.00138
X2	0.61703	-0.10112
X3	0.49899	-0.05083
X4	0.47995	0.12903
X5	0.03523	0.49182
X6	-0.04097	0.47610
X7	0.04314	0.43286
X8	-0.07363	0.58452

Variance Explained by Each Factor

Factor	Weighted	Unweighted
Factor1	1.52675593	1.06939360
Factor2	1.41007520	1.02705058

Final Communality Estimates and Variable Weights
Total Communality: Weighted = 2.936831 Unweighted = 2.096444

Variable	Communality	Weight
X1	0.19912149	1.24864087
X2	0.39095278	1.64189820
X3	0.25157813	1.33614951
X4	0.24699900	1.32799573
X5	0.24312507	1.32123679
X6	0.22835359	1.29597303
X7	0.18923091	1.23338999
X8	0.34708321	1.53154694

Non-identified 2-factor model

Start at rotated solution: Objective Function = 0.046133184?

Skippping a lot: The answer is *YES*.

Optimization Results

Iterations	7	Function Calls	11
Jacobian Calls	8	Active Constraints	0
Objective Function	0.046133184	Max Abs Gradient Element	4.6322427E-6
Lambda	1.110223E-14	Actual Over Pred Change	0.9919197308
Radius	0.036444217		

Skip some more: Loadings moved a bit from starting values, but not much.

Manifest Variable Equations with Estimates

X1	=	0.4453*F1	+	0.0280*F2	+	1.0000 delta1
Std Err		0.0572 lambda11		0.0503 lambda12		
t Value		7.7895		0.5574		
X2	=	0.6224*F1	+	-0.0603*F2	+	1.0000 delta2
Std Err		0.0605 lambda21		0.0434 lambda22		
t Value		10.2860		-1.3892		
X3	=	0.5013*F1	+	-0.0178*F2	+	1.0000 delta3
Std Err		0.0577 lambda31		0.0482 lambda32		
t Value		8.6856		-0.3704		
X4	=	0.4704*F1	+	0.1604*F2	+	1.0000 delta4
Std Err		0.0569 lambda41		0.0497 lambda42		
t Value		8.2649		3.2253		
X5	=	0.00275*F1	+	0.4931*F2	+	1.0000 delta5
Std Err		0.0477 lambda51		0.0594 lambda52		
t Value		0.0576		8.2989		
X6	=	-0.0722*F1	+	0.4723*F2	+	1.0000 delta6
Std Err		0.0484 lambda61		0.0588 lambda62		
t Value		-1.4925		8.0318		
X7	=	0.0145*F1	+	0.4348*F2	+	1.0000 delta7
Std Err		0.0498 lambda71		0.0587 lambda72		
t Value		0.2918		7.4112		
X8	=	-0.1120*F1	+	0.5784*F2	+	1.0000 delta8
Std Err		0.0446 lambda81		0.0611 lambda82		
t Value		-2.5132		9.4658		

Just show the unrotated and rotated loadings.

Factor Pattern

	Factor1	Factor2
X1	0.91601	0.22481
X2	0.92820	0.22462
X3	0.92296	0.22046
X4	0.90313	0.24962
X5	-0.22649	0.92123
X6	-0.25341	0.90484
X7	-0.23431	0.90476
X8	-0.26506	0.91071

Rotated Factor Pattern

	Factor1	Factor2
X1	0.94311	-0.01269
X2	0.95486	-0.01594
X3	0.94874	-0.01865
X4	0.93687	0.01456
X5	0.01237	0.94858
X6	-0.01780	0.93948
X7	0.00066	0.93461
X8	-0.02761	0.94810

And recall, these are the true parameter values:

```
/* True factor loadings (All communalities = .5^2 + .8^2 = 0.89) */
     L11 = .5; L12 =  -.8;
     L21 = .5; L22 =  -.8;
     L31 = .5; L32 =  -.8;
     L41 = .5; L42 =  -.8;
     L51 = .8; L52 = .5;
     L61 = .8; L62 = .5;
     L71 = .8; L72 = .5;
     L81 = .8; L82 = .5;
```