STA 2201 Assignment 5

You will be asked to hand this one in at the *beginning* of class on Tuesday March 2nd. For the questions requiring use of software, please attach a printout.

1. In ordinary normal regression, when $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{h}$ is false, the test statistic has a non-central F distribution with a non-centrality parameter we should write as ϕ_n, because it depends on the sample size.

   Express the non-centrality parameter as $\phi_n = n\delta_n^2$. Under what conditions does δ_n^2 converge to a constant δ? There are two cases you should consider, and a somewhat different mode of convergence in each case.

2. In ordinary normal regression, the statistic for testing $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{h}$ can be written
$$F_n^* = \left(\frac{n-r}{q}\right)\left(\frac{\widehat{a}_n}{1-\widehat{a}_n}\right),$$
   where
$$\widehat{a}_n = \frac{R_F^2 - R_R^2}{1 - R_R^2}.$$
   Show that under the conditions you gave in Question 1, $\widehat{a}_n$ converges to a constant a. This is Cohen's population effect size.

3. Remember $\widehat{\phi}_n = qF_n^* = n\widehat{\delta}_n^2$? Under the conditions you gave in Question 1, show that $\widehat{\delta}_n^2 \stackrel{P}{\rightarrow} \delta$. Use the Convergence Handout. By the way, there is a slightly updated version of the handout; it's in outline form, so it's easier to refer to 7a and so on.

4. In a test of the equality of two means with a standard F-test, we would like to be able to detect a difference of half a standard deviation with probability 0.80. Sample sizes are equal. This corresponds to what Cohen-style effect size a? Show your work. What unnecessary piece of information did I provide?

5. In ordinary normal regression, suppose that $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{h}$ is *true*. In this case, does $\widehat{a}_n$ converge to something meaningful? Answer Yes or No, and justify your answer.

   A good answer to this question will start with a derivation of the limiting distribution of the test statistic F_n^* as n goes to infinity. Remember , it's the ratio of two independent chi-squares, each divided by its degrees of freedom, a chisquare is the sum of independent squared standard normals, and so on. Use the Convergence Handout.

   By the way, I think I said something wrong about this in lecture, so don't let it influence your answer.

6. A psychologist comes to you and says "I am planning to do a oneway analysis of covariance with 6 experimental treatments and two covariates. I want to test a planned comparison contrasting the first treatment with the average of the last three. If the effect in question explains 15% of the remaining variance after correcting for the covariates, what sample size should I have?"

   Note that this question is expressed in the language that psychologists use to talk about statistics, and I know it may be a little ambiguous to you. You have to guess what is meant, but you can probably guess right. Assume you chatted with the psychologist and verified that $\alpha = 0.05$, and a power of 0.80 would be okay. My answer is $n = 55$.

7. Suppose you are planning a one-factor analysis of variance with 4 treatments, and your budget limits you to 15 cases per treatment. How big does R^2 have to be for you to reject H_0 at the 0.05 level?

8. For the psychologist with the analysis of covariance, what sample size is required to reject the null hypothesis if 15% of the remaining *sample* variation is explained? My answer is $n = 33$.