STA 2201 Assignment 2

There will be a brief quiz on this material at the *beginning of class* on Tuesday Jan. 27. Please bring your printouts to class. You may (or may not) be asked to hand them in. Other quiz questions will be very much like the ones below, but based on a different data set.

An experiment in dentistry seeks to test the effectiveness of a drug (HEBP) that is supposed to help dental implants become more firmly attached to the jaw bone. This is an initial test on animals. False teeth were implanted into the leg bones of rabbits, and the rabbits were randomly assigned to receive either the drug or a saline solution (placebo). Technicians administering the drug were blind to experimental condition.

Rabbits were also randomly assigned to be "sacrificed" after either 3, 6, 9 or 12 days. At that time, the implants were pulled out of the bone by a machine that measures force in newtons and stiffness in newtons/mm. For both of these measurements, higher values indicate more healing. A measure of "pre-load stiffness" in newtons/mm is also available for each animal. This may be another indicator of how firmly the false tooth was implanted into the bone, but it might even be a covariate. Nobody can seem to remember what "preload" means, so we'll ignore this variable for now.

The data are available in the file `bunnies.dat` (see course home page for a link). The variables are

- Identification code
- Time (3,6,9,12 days of healing)
- Drug (1=HEBP, 0=saline solution)
- Stiffness in newtons/mm
- Force in newtons
- Preload stiffness in newtons/mm

Consider a two-factor analysis of variance in which the factors are Drug and Time, and the outcome variable is Force. To establish notation, here is a table of the expected values about which we want to make inference. You can imagine an additional row and an additional column of *marginal means*, containing averages across columns and rows respectively,

	Healing Time in Days			
	3	6	9	12
Exp. Drug	μ_{11}	μ_{12}	μ_{13}	μ_{14}
Placebo	μ_{21}	μ_{22}	μ_{23}	μ_{24}

1. Before doing any computer work, please do the following questions. They are not to be handed in, but if you do them with understanding, you will have no trouble with the quiz.

(a) The table below shows a dummy variable coding setup not mentioned in class, though you may have encountered it elsewhere (it's called "effect coding"). Fill in the last column with

$$E[Y] = \beta_0 + \beta_1 d_1 + \beta_2 h_1 + \beta_3 h_2 + \beta_4 h_3 + \beta_5 d_1 h_1 + \beta_6 d_1 h_2 + \beta_7 d_1 h_3$$

Drug	Time	d_1	h_1	h_2	h_3	$E[Y]$
1	3	1	1	0	0	$\mu_{11} =$
1	6	1	0	1	0	$\mu_{12} =$
1	9	1	0	0	1	$\mu_{13} =$
1	12	1	-1	-1	-1	$\mu_{14} =$
0	3	-1	1	0	0	$\mu_{21} =$
0	6	-1	0	1	0	$\mu_{22} =$
0	9	-1	0	0	1	$\mu_{23} =$
0	12	-1	-1	-1	-1	$\mu_{24} =$

(b) Suppose you want to test the null hypothesis of no main effect for Drug. That is, you want to test whether the two marginal means for Drug are equal. Express the null hypothesis in terms of β values. Show your work. Just to get you started and to make this type of question more clear, I'll tell you the answer: $H_0 : \beta_1 = 0$. The "show your work" part consists of establishing that

$$\frac{\mu_{11} + \mu_{12} + \mu_{13} + \mu_{14}}{4} = \frac{\mu_{21} + \mu_{22} + \mu_{23} + \mu_{24}}{4} \Leftrightarrow \beta_1 = 0\,.$$

(c) What is $E[Y]$ in the reduced model of the test for Drug main effect?

(d) The null hypothesis of no main effect for Drug may be written $\mathbf{C}\boldsymbol{\beta} = \mathbf{h}$. What is $\mathbf{C}$? What is $\mathbf{h}$?

(e) Express the nll hypothesis of no main effect for healing time in terms of $\beta values$. Show your work.

(f) What is $E[Y]$ in the reduced model of the test for Time main effect?

(g) The null hypothesis of no main effect for Time may be written $\mathbf{C}\boldsymbol{\beta} = \mathbf{h}$. What is $\mathbf{C}$? What is $\mathbf{h}$?

(h) Express the nll hypothesis of Drug by Healing Time interaction, in terms of $\beta values$. Show your work.

(i) What is $E[Y]$ in the reduced model of the test for the interaction?

(j) The null hypothesis of Druf by Time interaction may be written $\mathbf{C}\boldsymbol{\beta} = \mathbf{h}$. What is $\mathbf{C}$? What is $\mathbf{h}$?

2. Now make a table showing a dummy coding scheme with no intercept and 12 indicator dummy variables, one for each treatment combination (this is called "cell means" coding). Give $E[Y]$ for each treament combination.

3. Using your cell means coding scheme, Give the **C** matrix for each test indicated below.

   (a) Main effect for Time.

   (b) Drug by Time interaction.

   (c) Departure from linearity of the margainal means for time. (Think about it. My **C** matrix has 2 rows.)

4. Now read the data into S and do the following.

   (a) Use `table` to find out how many rabbits are in each experimental condition.

   (b) Using `tapply`, (see updated overheads on the Web if necessary) make a table of cell means for the dependent variable Force.

   (c) Test both main effects and the interaction. In plain language, what would you say to the dental researchers who collected the data? What I mean is, did the drug work?

   (d) Try to test the hypothesis of departure from linearity in the marginal means for Time. I will be impressed if you can do this. Don't worry if you get stuck.