STA 2201 S 2004 Assignment 1

Due Tuesday Jan 20th at the *beginning* of class.

1. This first item is just to warm you up to exact likelihood ratio testing and S programming with an elementary problem that presents no special difficulties.

   (a) Let $X_1, \ldots, X_n$ be a random sample from a Normal(μ, σ^2) distribution where σ^2 is *known*. Derive an exact likelihood ratio test of $H_0 : \mu = \mu_0$ versus the alternative that $\mu \neq \mu_0$. Show your work.

   (b) Write an S function to calculate your test. The function should accept the hypothesized mean μ_0, the known variance σ^2, and a data vector as input, and return a vector consisting of the sample mean, the value of the test statistic, and a p-value.

   (c) Run your function on the following data, with $\mu_0 = 20$ and $\sigma^2 = 15$:
   `21 18 12 17 18 20 14 23 17 22 23 22 27 19 21 21 25 17 21 14 15 12 10 19 18 26 22 23 16 19 19 17 26 15 18 17 19 21 17 27 .`

2. Now you'll do something a little more interesting, though it's still elementary. Let $X_1, \ldots, X_{n_1}$ and $Y_1, \ldots, Y_{n_2}$ be independent random samples from Normal(μ_1, σ_1^2) and Normal(μ_2, σ_2^2) distributions, respectively. If it's not stored somewhere your head, find an elementary Statistics text and locate the traditional F-test for $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative that they are not equal.

   (a) Attempt to derive the traditional test as an exact likelihood ratio test. Show your work. Near the end, you will be forced to stop calculating and wave your hands in order to arrive at the traditional test. Please indicate exactly where precise mathematical work stops and the hand waving begins. But proceed, arriving finally at the traditional test. You do not need to derive any distributions; just cite a few well-known results along the way.

   (b) Write an S function to calculate the traditional test. Your function should accept two numeric vectors of sample data as input. The output should be a vector consisting of the variance of Sample One, the variance of Sample Two, the F-statistic and a p-value.

   (c) Run your function on the following data:
   X: `13.9 8.0 9.5 9.0 10.9 9.0 9.7 9.7 9.0 12.8 13.3 8.8 12.3 10.6 13.0 8.1 9.8 9.9 11.7 9.9 12.3`
   Y: `10.0 9.9 15.7 11.9 14.9 14.5 12.0 11.1 9.2 11.3 12.9 8.6 11.1 12.0 11.0 13.0 11.1 13.9 10.1 8.9 10.7 11.4 8.5 12.2 12.2 12.1 12.1 15.5 10.7 5.4 13.4 14.2 16.7 14.0 10.6 10.3 8.9 14.4 12.3 13.7 9.3 15.0 7.1 9.7 9.2`

What you hand in: For each problem, your hand-written calculations, a printout listing your S function, and another printout showing inpput and output. If the last two items are on the same physical piece of paper, that's fine.

What you may have learned: Applied Statistics often involves movement back and forth between paper-and-pencil calculations and computer work. Some hand-waving is allowed, but you are supposed to do it sparingly and show good taste.