Indicator functions: This notation is not in the text!

Let A be a set of real numbers. Then the **indicator function** for A is defined by

$$I_A(x) = I\{x \in A\} = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

Ex. $I\{x \geq 0\} = I_{[0,\infty)}(x)$ $\quad$ $I\{x=1,2,3\} = I_{\{1,2,3\}}(x)$

$I\{a < x \leq b\} = I_{(a,b]}(x)$ $\quad$ $I\{x=0,1, \ldots\} = I_{\{0,1, \ldots\}}(x)$

Two important properties of indicator functions are $I_A(x)\, I_B(x) = I_{A \cap B}(x)$ and if g(x) is a real valued function,

$$g(x)\, I_A(x) = \begin{cases} g(x) & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

Def. The **support** of a discrete random variable is the set of x values for which $P(X=x) > 0$.

In this class, probability density functions and probability mass functions will always be defined for all real x, and will include indicators for their support.

For example, where the book might write $\quad f(x) = \begin{cases} \frac{x}{6} & \text{for } x = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$

we will write $f(x) = \frac{x}{6}\, I\{x = 1,2,3\}$.

The exponential density could be written $f(x) = \theta\, e^{-\theta x}\, I\{x>0\}$