Name Jerry

Student Number

STA 312 f2023 Quiz 3

Let $X_1, \ldots, X_n$ be a random sample (that is, independent and identically distributed) from a Poisson distribution with parameter $\lambda > 0$. You already know that the maximum likelihood estimate is $\widehat{\lambda} = \overline{X}$. We want to test $H_0 : \lambda = \lambda_0$ versus $H_1 : \lambda \neq \lambda_0$ with a large-sample likelihood ratio test. For this problem, the subset of the parameter space specified by the null hypothesis is a single point: $\Theta_0 = \{\lambda_0\}$.

1. (7 points) Write down and simplify the G^2 test statistic. A variety of "simplified" answers can be correct. Your final answer is a formula. **Circle it**.

$$G^2 = -2 \log \frac{L(\hat{\theta}_0)}{L(\hat{\theta})} \quad \left(L = \prod_{i=1}^n \frac{e^{-\lambda} \lambda^{x_i}}{x_i!} = \frac{e^{-n\lambda} \lambda^{\Sigma x_i}}{\prod_{i=1}^n x_i!} \right)$$

$$= -2 \log \frac{e^{-n\lambda_0} \lambda_0^{n\bar{x}} \Big/ \cancel{\prod_{i=1}^n x_i!}}{e^{-n\bar{x}} \bar{x}^{n\bar{x}} \Big/ \cancel{\prod_{i=1}^n x_i!}}$$

$$= -2\left(-n\lambda_0 + n\bar{x} \log \lambda_0 - \left[-n\bar{x} + n\bar{x} \log \bar{x}\right]\right)$$

$$= -2\left(-n\lambda_0 + n\bar{x} \log \lambda_0 + n\bar{x} - n\bar{x} \log \bar{x}\right)$$

$$= 2n\left(\lambda_0 - \bar{x} \log \lambda_0 - \bar{x} + \bar{x} \log \bar{x}\right)$$

$$= 2n\left(\lambda_0 - \bar{x} + \bar{x}(\log \bar{x} - \log \lambda_0)\right)$$

This is fine, or

$$= 2n\left(\lambda_0 - \bar{x} + \bar{x} \log\left(\frac{\bar{x}}{\lambda_0}\right)\right)$$

Repeating, $G^2 = 2n\left(\lambda_0 - \bar{x} + \bar{x}(\log \bar{x} - \log \lambda_0)\right)$

Guidelines:

- If there is no answer to part 1 or the answer can't be computed, Part 2 gets zero.
- They don't have to show this much detail
- If answer to part 1 is wrong, carry it forward
- Negative answers without comment get zero on part (2): −3

2. (3 points)

(a) A random sample of size $n = 49$ yields a sample mean of 4.2 and a sample standard deviation of 2.14. We want to test $H_0 : \lambda = 3$. Calculate your G^2 statistic. Show a little work. The answer is a number. **Circle your answer.**

$$G^2 = 2 \times 49\left(3 - 4.2 + 4.2(\log 4.2 - \log 3)\right)$$
$$= 98\left(-1.2 + 4.2(1.435 - 1.0986)\right)$$
$$= 98(1.41 - 1.2) = 98 \times 0.21 = 20.58$$

(b) What are the degrees of freedom? The answer is a number.

Rounding error is OKAY.

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(c) The critical chi-squared value at $\alpha = 0.5$ is $1.96^2 = 3.84$. Do you reject H_0? Answer Yes or No.

Yes

Answer to (c) should be consistent with (a).