Name Jerry

Student Number _____

STA 312 f2023 Quiz 2

1. (5 points) Let $X_1, \ldots, X_n$ be a random sample from a geometric distribution; see formula sheet. Find the maximum likelihood estimate of θ. Show your work and circle your final answer. *You do not need to bother with the second derivative test.*

* Bold

$$\ell(\theta) = \log \prod_{i=1}^{n} (1-\theta)^{x_i - 1}\theta = \log\left((1-\theta)^{\sum x_i - n}\theta^n\right)$$

$$= n\log\theta + (\textstyle\sum x_i - n)\log(1-\theta)$$

$$\ell'(\theta) = \frac{n}{\theta} + \frac{(\sum x_i - n)}{1-\theta}(-1) \stackrel{\text{set}}{=} 0$$

$$\Rightarrow \frac{n}{\theta} = \frac{\sum_{i=1}^{n} x_i - n}{1-\theta}$$

$$\Rightarrow \theta\textstyle\sum x_i - n\theta = n - n\theta$$

$$\Rightarrow \theta = \frac{n}{\sum_{i=1}^{n} x_i}, \text{ so}$$

$$\hat{\theta} = \frac{n}{\sum_{i=1}^{n} x_i} \text{ or } \hat{\theta} = 1/\bar{x}$$

Either one

2. For Question 7 of Assignment 2, you analyzed numerical data from a distribution with density $f(x|\pi) = \pi e^{-\pi/x}\frac{1}{x^2}$ for $x > 0$, where the parameter $\pi > 0$.

   (a) (2 points) In the space below, write $\widehat{\pi}$, the maximum likelihood estimate of π. The answer is a number from your printout. On your printout, circle the number and write "Question 2a" beside it.

$\hat{\pi} = 2.74$

   (b) (3 points) You found a 95% confidence interval for π. In the space below, write the lower and upper confidence limits. The answer is a pair of numbers from your printout. On your printout, circle the numbers and write "Question 2b" beside them.

Confidence interval is

(2.43, 3.05)

Please attach the printout with your answers to Question 2 of this quiz (Question 7 of the assignment). Make sure your name and student number are written on the printout.

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R version 3.6.3 (2020-02-29) -- "Holding the Windsock"
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[Workspace restored from /Users/brunner/.RData]
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> x = scan("http://www.utstat.toronto.edu/brunner/data/legal/inversegamma.data.txt")
Read 300 items
> n = length(x)
> pihat = 1/mean(1/x); pihat
[1] 2.740541
> se_pihat = sqrt(pihat^2/n); se_pihat
[1] 0.1582252
> CI = c(pihat-1.96*se_pihat, pihat+1.96*se_pihat); CI
[1] 2.430420 3.050662
> Z = (pihat-3.14159)/se_pihat; Z
[1] -2.534672
> pvalue = 2 * (1-pnorm(abs(Z))); pvalue
[1] 0.01125527
>
>
```

Question 2a

Question 2b