

Survival and Hazard Functions¹

STA 312 Fall 2023

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Background Reading

Chapter Two in *Applied Survival Analysis Using R*

Survival Function

Let T be a continuous random variable with $P(T > 0) = 1$. It will represent time to failure. The *survival function* is the probability of lasting past time t .

$$\begin{aligned} S(t) &\stackrel{def}{=} P(T > t) \\ &= 1 - F(t) \\ &= \int_t^{\infty} f(x) dx \end{aligned}$$

The hazard function $h(t)$

- Idea: The probability of dying at time t given that you have lived this long.
- But the probability of dying at exactly time t is zero.
- So consider the probability of dying in the next *instant* following t , given that you have lived to time t .
- The meaning of *instant* is a tiny interval of time, length Δ .

$$P(t < T < t + \Delta | T > t)$$

- This clearly depends on the length of the time interval Δ . So make the conditional probability *relative* to the length of time Δ .

$$\frac{P(t < T < t + \Delta | T > t)}{\Delta}$$

- To capture the idea of a *really* tiny interval, let $\Delta \rightarrow 0$. This leads to the definition

$$h(t) \stackrel{def}{=} \lim_{\Delta \rightarrow 0} \frac{P(t < T < t + \Delta | T > t)}{\Delta}$$

The Hazard Function

$$h(t) \stackrel{\text{def}}{=} \lim_{\Delta \rightarrow 0} \frac{P(t < T < t + \Delta | T > t)}{\Delta}$$

where $\Delta > 0$.

It will be shown that

$$h(t) = \frac{f(t)}{S(t)}$$

The *Cumulative* Hazard Function

$$H(t) = \int_0^t h(x) dx$$

$\stackrel{\text{show}}{=} -\log S(t)$

So that

$$S(t) = e^{-\int_0^t h(x) dx}$$

Density, Cumulative Distribution Function, Survival Function and Hazard Function are all Equivalent

$$f(t) = F'(t)$$

$$F(t) = \int_{-\infty}^t f(x) dx$$

$$S(t) = 1 - F(t)$$

$$h(t) = \frac{f(t)}{S(t)}$$

$$S(t) = e^{-\int_0^t h(x) dx}$$

- With one function, it is easy to obtain the others.
- We will focus on the hazard function.

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<http://www.utstat.toronto.edu/brunner/oldclass/312f23>