UNIVERSITY OF TORONTO

Faculty of Arts and Science

Term Test, October 7th, 2008

ACT 460 / STA 2502

DURATION - 150 **minutes**

EXAMINER: Prof. S. Jaimungal

COURSE CODE (**circle one**): ACT 460 STA 2502

LAST NAME: ____

FIRST NAME: ____

STUDENT #: ____

Each question is worth 10 points

– NOT ALL QUESTIONS ARE OF THE SAME DIFFICULTY .

Please write clearly!

1 *[10]*	2 *[10]*	3 *[10]*	4 *[10]*	5 *[10]*	6 *[10]*	7 *[10]*	Total *[70]*

1. *[10]* Please indicate true or false. **no explanations required**

   -1 **for incorrect answer,** $+2$ **for correct answer,** 0 **for blank answer** .

   (a) [T] [F]

   If an economy admits a strategy which costs nothing and at a future time has strictly positive values with probability greater than zero, then this economy admits an arbitrage.

   (b) [T] [F]

   The price of a put option always increases with volatility.

   (c) [T] [F]

   In a one-period economy, the risk-neutral branching probabilities are always uniquely determined.

   (d) [T] [F]

   If S_t is the price of a traded stock, then in the Black-Scholes economy, the risk-neutral expected rate of return of S_t^2 is equal to $2\,r + \sigma^2$.

   (e) [T] [F]

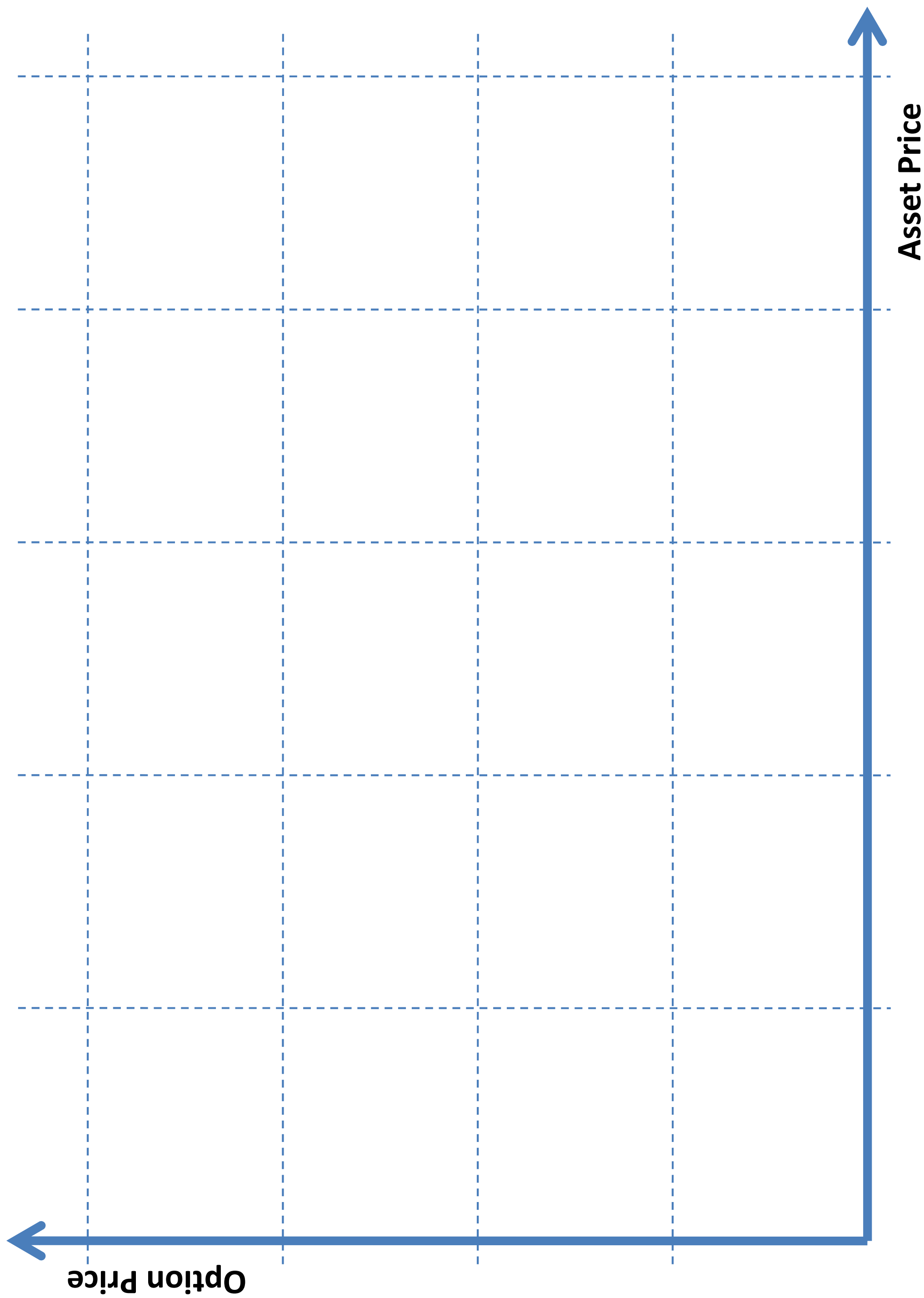
   A put option struck at \$100 trades at \$10, while a put option struck at \$110 trades at \$11. Both puts have the same time to maturity. This economy admits an arbitrage.

   [Hint: Consider 11 units of the first put and 10 units of the second put]

2. Sketch the option price as a function of the current spot-level for maturities of $T = 0$, $T = 1$ month and $T = 1$ year for

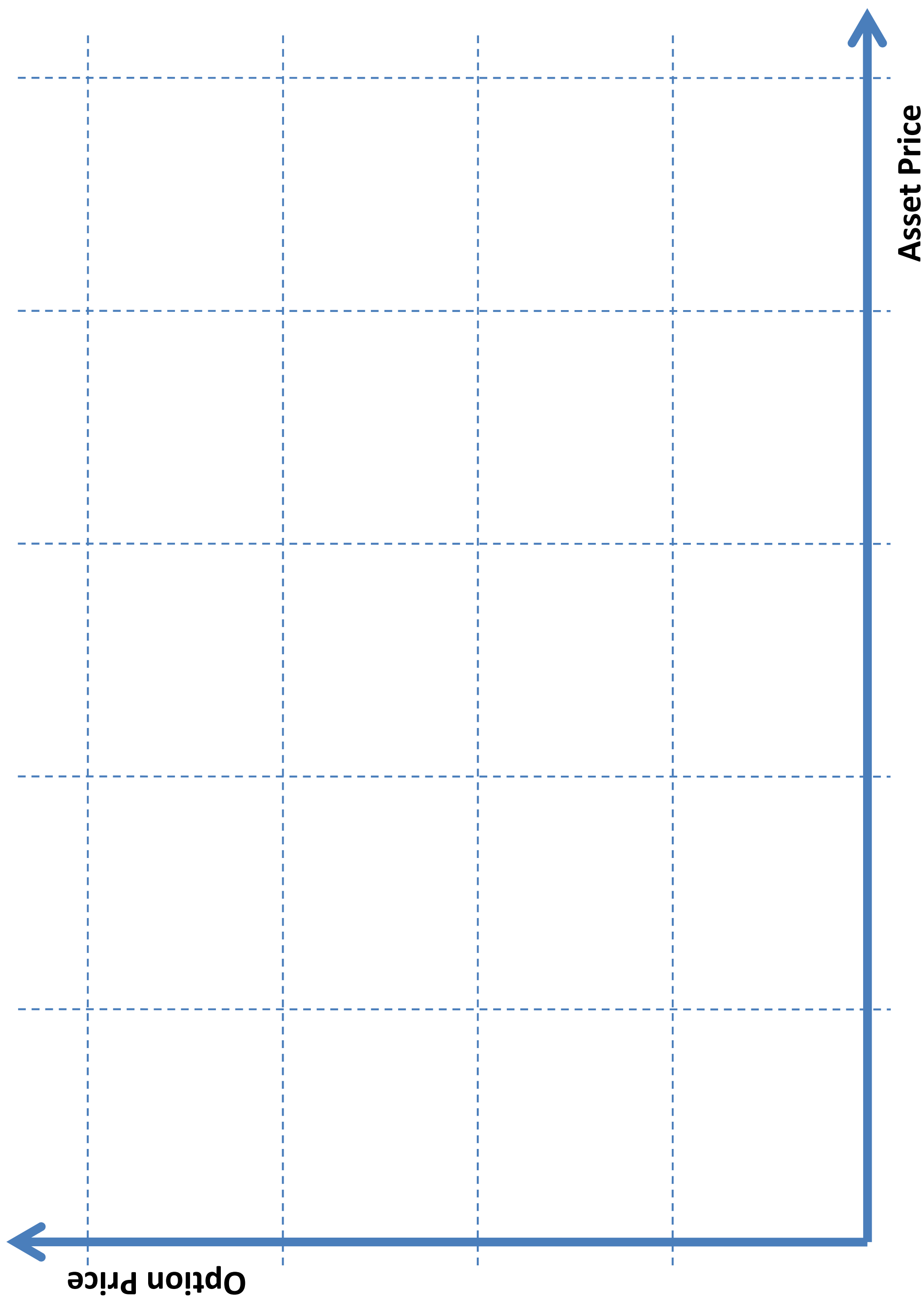
(a) *[5]* call option

[draw the three curves on the same graph, clearly label them and any interesting points.]

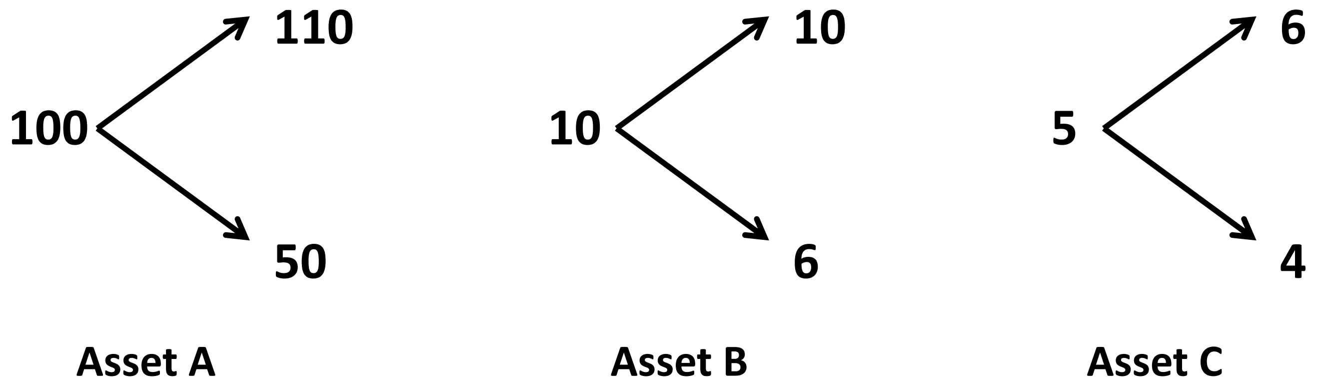


(b) *[5]* bear spread option. This option can be viewed as a long put struck at K_2 and a short put struck at K_1 $(0 < K_1 < K_2 < \infty)$

[draw the three curves on the same graph, clearly label them and any interesting points..]



3. *[10]* Consider an economy with the three traded assets below. Construct an arbitrage strategy.



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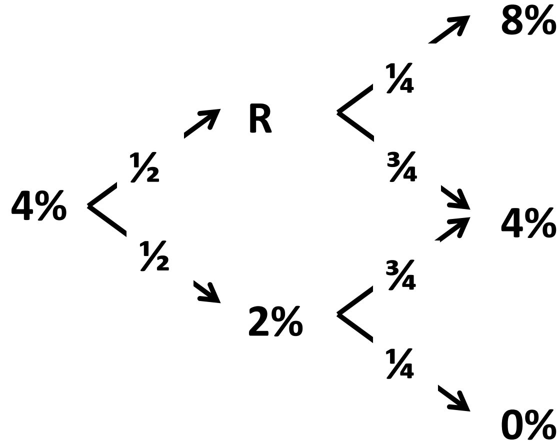
4. Consider the interest rate tree shown in the diagram below. The rates correspond to effective discounting – e.g. discounting over the first period is $1/(1+0.04)$. The probabilities shown are risk-neutral probabilities.



(a) *[6]* Consider a 2-year coupon bearing bond with coupons of \$5 paid every year and notional of \$100. Determine the rate R such that the bond is valued at par (i.e. has value \$100 now).

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(b) *[4]* Determine the price of a call option maturing at $t = 1$ written on the coupon bearing bond with strike equal to today's price of the bond. Note: the option holder will not receive the coupon due at $t = 1$.

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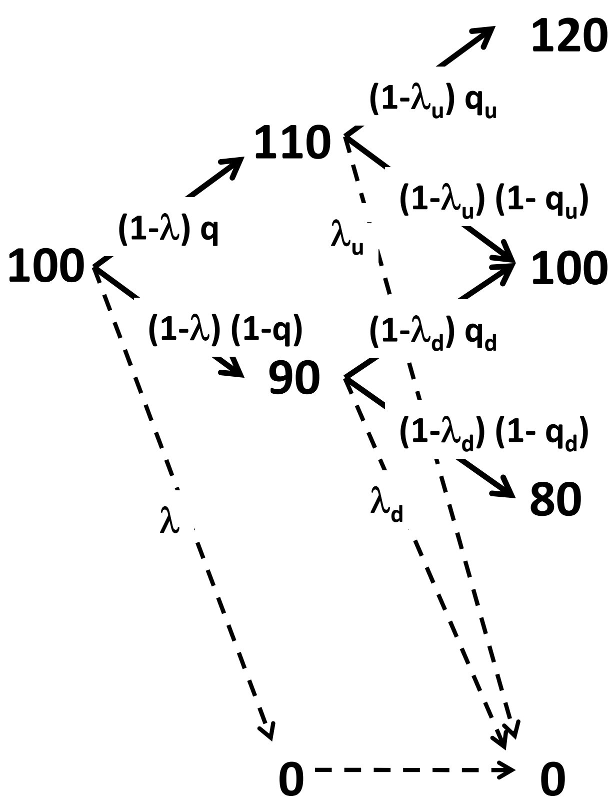
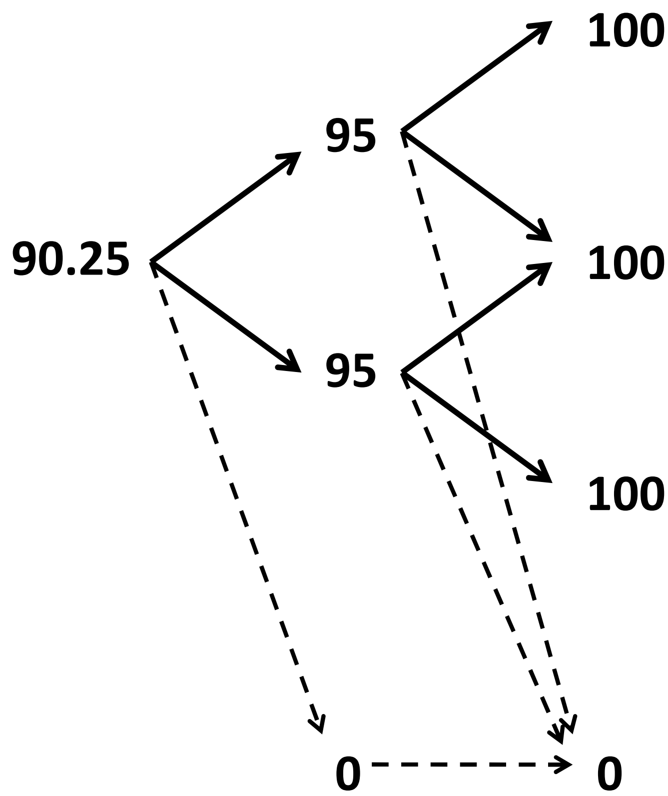
5. Consider an economy with a defaultable stock and bond (interest rates are 0 in all states of the world):



Defaultable Stock price tree



Defaultable Bond price tree

(a) *[5]* Show that the risk-neutral default probabilities shown in the diagram are as follows:

$$\lambda = \lambda_d = \lambda_u = \frac{1}{20}, \qquad q = \frac{29}{38}\ , \quad q_d = \frac{14}{19}\ , \quad q_u = \frac{15}{19}\ .$$

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(b) *[5]* Value an American put option with the stock as underlier, strike of 100 and maturity of two periods.

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6. (a) *[5]* Assuming the Black-Scholes model, derive an expression for a "forward start digital call option". A forward start digital call is an option which pays 1 at the maturity date T if the stock price at time T is larger than the stock price at time U. ($U < T$) Write your answer in terms of $\Phi(x) := \mathbb{Q}(Z < x)$ where Z is a standard normal random variable under the measure $\mathbb{Q}$.

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(b) *[5]* Assuming the Black-Scholes model, derive an expression for a contingent claim which pays the geometric average of the asset price at two points in time. That is, the claim pays $(S_{T_1} S_{T_2})^{\frac{1}{2}}$ at maturity T_2 where $0 < T_1 < T_2$.

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7. Suppose you model stock prices in a CRR like fashion. However, you assume that

$$S_n = S_{n-1} \exp\{(r - \tfrac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\, x_n\}$$

where $x_1, x_2, \ldots$ are iid r.v. with $\mathbb{P}(x_1 = +1) = p$ and $\mathbb{P}(x_1 = -1) = 1 - p$.

(a) *[5]* Prove that if we force

$$\mathbb{E}^{\mathbb{P}}[S_T] = S_0\, e^{\mu T} ,$$
$$\mathbb{V}^{\mathbb{P}}[\ln(S_T/S_0)] = \sigma^2\, T$$

in the limit as $\Delta t \downarrow 0$ while $T = n\Delta t$ is held fixed. Then,

$$p = \frac{1}{2}\left(1 + \frac{\mu - r}{\sigma}\sqrt{\Delta t}\right) + O(\Delta t) \ .$$

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(b) *[5]* Prove that, in the limit as $\Delta t \downarrow 0$ while $T = n\Delta t$ is held fixed, the risk neutral probability in this model is (with constant rate of interest r)

$$q = \frac{1}{2} + O(\Delta t) \ .$$

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