

1. (Knight, 2000 Ch. 5.6; Owen, 1988). Suppose Y_1, \dots, Y_n are independent and identically distributed from an unknown distribution function F . To estimate F we restrict attention to distributions putting positive probability mass only at the points Y_1, \dots, Y_n , assumed distinct. Knight defines the non-parametric log-likelihood function for $F(\cdot)$ as

$$L(p) = L(p_1, \dots, p_n) = \prod_{i=1}^n p_i, \quad p_i \geq 0, \Sigma p_i = 1,$$

where p_i is the probability mass at Y_i .

- (a) Show that $L(p)$ (or equivalently $\ell(p) = \log L(p)$) is maximized at $\hat{p}_i = 1/n$.
- (b) Suppose that $\mu = E(Y_i) = \int y dF(y)$ is the parameter of interest, with $F(\cdot)$ as a nuisance parameter. The profile likelihood is obtained by maximizing

$$L(p_1, \dots, p_n), \text{ subject to } p_i \geq 0, \Sigma p_i = 1, \Sigma p_i Y_i = \mu,$$

where there is now an additional constraint on the vector p . Show that the solution to the maximization problem is given by

$$\begin{aligned} \hat{p}_i(\mu) &= \frac{1}{n} \frac{1}{1 + \lambda(Y_i - \mu)}, \text{ where } \lambda \text{ solves} \\ 0 &= \frac{1}{n} \sum_{i=1}^n \frac{Y_i - \mu}{1 + \lambda(Y_i - \mu)}. \end{aligned}$$

2. Choose a paper for your report and presentation, and provide the complete citation and a one-sentence description of the paper.

You should plan for a 15 minute presentation followed by 5 minutes of questions. The presentation can be either on slides or presented live on a tablet/ipad. My guideline for number of slides is one per minute. Your report should be between five and ten pages (five is enough), and include:

- (a) Complete citation

- (b) One paragraph overview of the main problem and results (without quoting the abstract).
- (c) A paragraph setting the work in context of the literature. (Again, not quoted directly from the paper.)
- (d) One or two sections outlining the techniques used to get from problem to results. For some papers this might mean quoting the most important theorem(s) and summarizing the key idea of the proof(s); for others it might be describing a new model that is proposed and explaining the key new features of this.
- (e) Two or three paragraphs providing your assessment of the paper and the work: was it interesting? was the paper well written? does it suggest further work? Is there an application suggested, and if so, did you find it convincing? How does it tie into topics discussed in this course?