

1. SM 4.9.2 Let $\eta(\theta)$ be a 1-1 transformation of θ , and consider a model with log-likelihoods $\ell(\theta)$ and $\ell^*(\eta) = \ell(\theta(\eta))$ in the two parametrizations respectively. Assume that $\ell(\cdot)$ has a unique maximum at which the score equation is satisfied.

Show that the Fisher information transforms as

$$i^*(\eta) = \frac{\partial \theta^T}{\partial \eta} i(\theta) \frac{\partial \theta}{\partial \eta},$$

and that a similar equation holds for observed information $j(\hat{\theta})$, but not for the observed information function $j(\theta)$.

2. Suppose that y_1, \dots, y_n are independent, identically distributed random variables from an exponential family distribution

$$f(y_i; \theta) = \exp\{\theta^T s(y_i) - c(\theta) - d(y_i)\}.$$

Show that the distribution of $S = \sum_{i=1}^n s(y_i)$ follows an exponential family distribution, with log-likelihood function

$$\ell(\theta; s) = \theta^T s - nc(\theta),$$

and that the maximum likelihood estimate of θ is given by

$$E_{\hat{\theta}}(S) = s.$$

3. This is a journal reading exercise.
- (a) Find a paper from 2020 or 2021 in one of the theoretical statistics journals (e.g. *Annals of Statistics*, *Biometrika*, *JASA Theory & Methods*, *JRSS B*, *Bernoulli*) with “likelihood” as one of the key words. Explain how likelihood is used in the paper.
 - (b) Find a paper from 2020 or 2021 in one of the applied statistics journals (e.g. *Annals of Applied Statistics*, *Biometrics*, *JASA applications*, *JRSS C*) with “likelihood” as one of the key words. Explain briefly how likelihood is used in the paper.