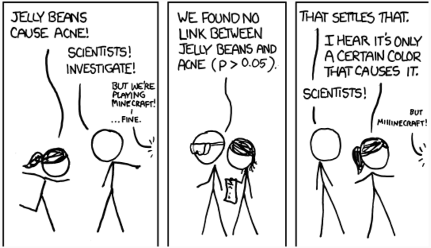
Mathematical Statistics II

STA2212H S LEC9101

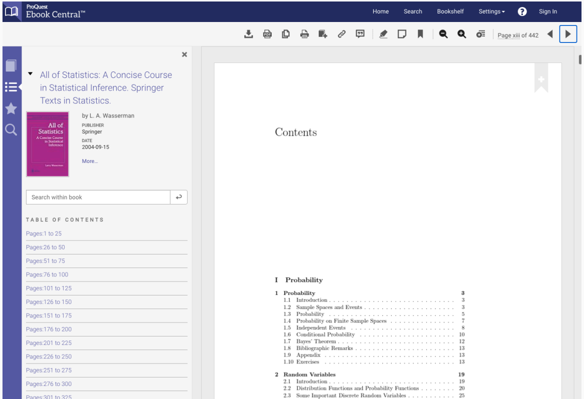
Week 3

January 27 2021



Recap

- Quasi-Newton; E-M algorithm — optimization
- Formal theory of testing: H_0, H_1, R, T, Type I, Type II error, β, α — hypothesis testing
- *p*-values
- Wald test, likelihood ratio test
- Chapter numbering AoS link



... Recap

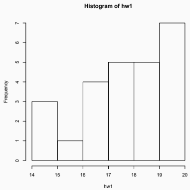
- Null and alternative hypothesis: $H_0 : \theta \in \Theta_0$; $H_1 : \theta \in \Theta_1$, $\quad \Theta_0 \cup \Theta_1 = \Theta$
- Rejection region: $R \subset \mathcal{X}$; if $\boldsymbol{x} \in R$ “reject” H_0
- Test statistic and critical value: $R = \{x \in \mathcal{X} : t(x) > c\}$ $\qquad$ c to be chosen
- Type I and Type II error: $\Pr\{t(X) > c \mid \theta \in \Theta_0\}$, $\qquad \Pr\{t(X) \leq c \mid \theta \in \Theta_1\}$
- Power and Size: $\beta(\theta) = \Pr_\theta(X \in R)$ $\qquad \alpha = \sup_{\theta \in \Theta_0} \beta(\theta)$
- Optimal tests: among all level-α tests, find that with the highest power under H_1

level-α means size $\leq \alpha$

Today

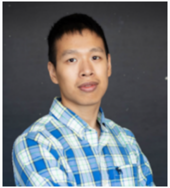
Start Recording

1. Homework
2. Choosing test statistics – NP lemma; score, Wald, LRT
3. Significance testing
4. Goodness-of-fit tests



- February 1 3.00 – 4.00 Joanna Mills Flemming
- “Statistical Tools for Better Understanding Marine Life” Link
- January 28 1.00 pm EST Linbo Wang
  CANSSI National Seminar Series
  “The promises of multiple outcomes”

Data Science and Applied Research Series



Choosing test statistics

1. Context

2. Optimal choice – Neyman-Pearson Lemma

3. Pragmatic choice – likelihood-based statistics

Choosing test statistics

Choosing test statistics

Example: Likelihood inference

Example: Likelihood inference

$X_1, \dots, X_n$ i.i.d. $f(x;\theta)$; $\hat{\theta}(X_n)$ is maximum likelihood estimate. From last week:

$$(\hat{\theta} - \theta)/\widehat{\text{se}} \dot{\sim} N(0,1)$$

To test $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$ we could use

$$W = W(X_n) = (\hat{\theta} - \theta_0)/\widehat{se},$$

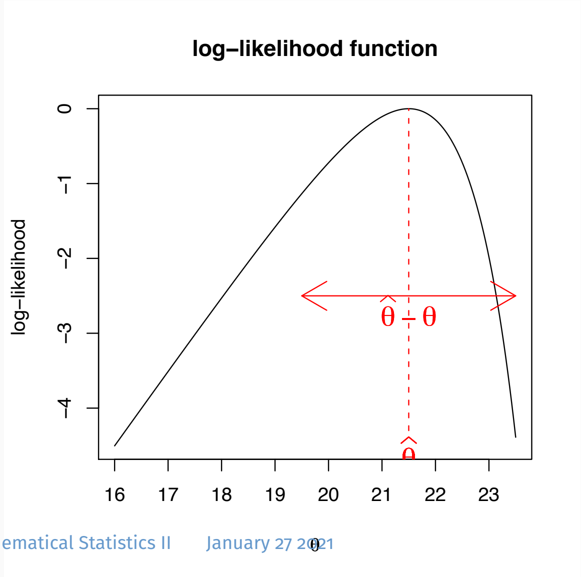
The critical region will be $\{\boldsymbol{x} : |W(\boldsymbol{x})| > z_{\alpha/2}\}$, i.e. “reject” H_0 when $|W| \geq z_{\alpha/2}$

This test has approximate size α:

$$\Pr(|W| > z_{\alpha/2}) \doteq \alpha.$$

Power? See Figure 10.1 and Theorem 10.6

... likelihood inference



Likelihood ratio tests

AoS 10.6, SM §4.5.1,2; §7.3 (p.330, 346)

Likelihood ratio tests

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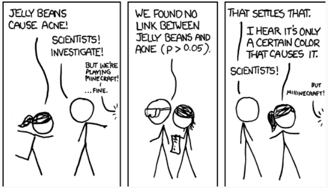
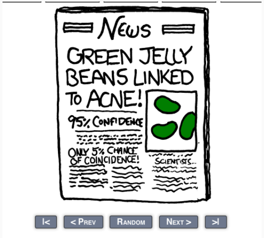
“p-hacking”





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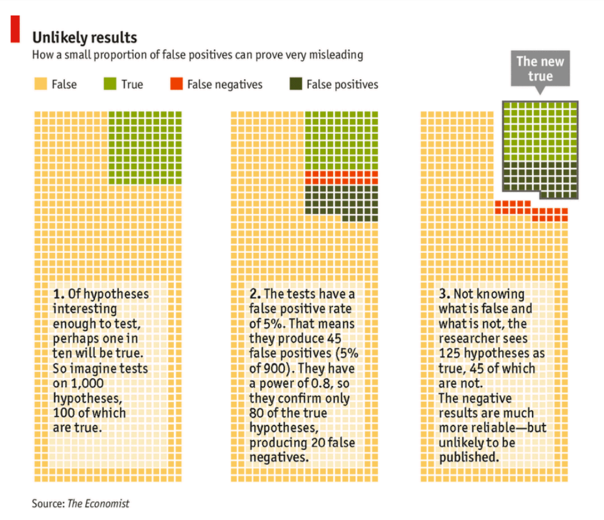

Encyclopedia article

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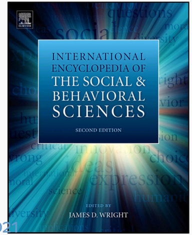

Goodness-of-fit tests

$\longrightarrow$ Monash and Face Masks