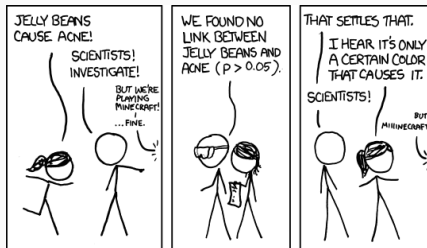


Mathematical Statistics II

STA2212H S LEC9101

Week 3

January 27 2021



Recap

- Quasi-Newton; E-M algorithm

optimization

- Formal theory of testing: H_0 , H_1 , R , T , Type I, Type II error, β , α

hypothesis testing

- p -values

- Wald test, likelihood ratio test

- Chapter numbering AoS link

The screenshot shows the 'All of Statistics' textbook interface on the Ebook Central platform. The top navigation bar includes 'Home', 'Search', 'Bookshelf', 'Settings', and 'Sign In'. The main content area displays the 'Contents' page, which lists the chapters and their page ranges. The left sidebar shows the book's title, author (L. A. Wasseman), publisher (Springer), and a search bar. The right sidebar shows the current page number (xii of 442).

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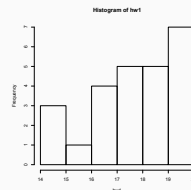
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... Recap

- Null and alternative hypothesis: $H_0 : \theta \in \Theta_0; H_1 : \theta \in \Theta_1, \quad \Theta_0 \cup \Theta_1 = \Theta$
- Rejection region: $R \subset \mathcal{X}$; if $\mathbf{x} \in R$ “reject” H_0
- Test statistic and critical value: $R = \{x \in \mathcal{X} : t(x) > c\}$ c to be chosen
- Type I and Type II error: $\Pr\{t(X) > c \mid \theta \in \Theta_0\}, \quad \Pr\{t(X) \leq c \mid \theta \in \Theta_1\}$
- Power and Size: $\beta(\theta) = \Pr_{\theta}(X \in R) \quad \alpha = \sup_{\theta \in \Theta_0} \beta(\theta)$
- Optimal tests: among all level- α tests, find that with the highest power under H_1
level- α means size $\leq \alpha$

1. Homework
2. Choosing test statistics – NP lemma; score, Wald, LRT
3. Significance testing
4. Goodness-of-fit tests



- February 1 3.00 – 4.00 Joanna Mills Flemming
- “Statistical Tools for Better Understanding Marine Life” [Link](#)
- January 28 1.00 pm EST Linbo Wang
CANSSI National Seminar Series
“The promises of multiple outcomes”

Data Science and Applied Research Series



Choosing test statistics

1. Context
2. Optimal choice – Neyman-Pearson Lemma
3. Pragmatic choice – likelihood-based statistics

Choosing test statistics

Choosing test statistics

Example: Likelihood inference

Example: Likelihood inference

X_1, \dots, X_n i.i.d. $f(x; \theta)$; $\hat{\theta}(X_n)$ is maximum likelihood estimate. From last week:

$$(\hat{\theta} - \theta)/\widehat{\text{se}} \sim N(0, 1)$$

To test $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$ we could use

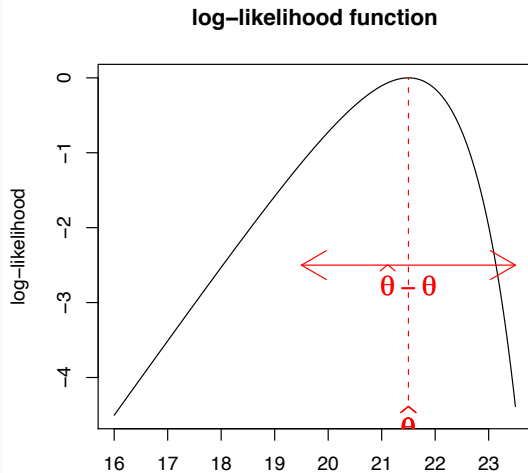
$$W = W(X_n) = (\hat{\theta} - \theta_0)/\widehat{\text{se}},$$

The critical region will be $\{\mathbf{x} : |W(\mathbf{x})| > z_{\alpha/2}\}$, i.e. “reject” H_0 when $|W| \geq z_{\alpha/2}$

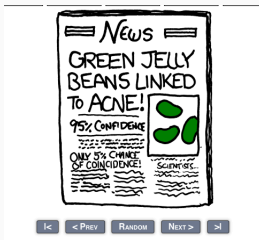
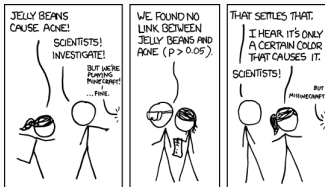
This test has approximate size α :

$$\Pr(|W| > z_{\alpha/2}) \doteq \alpha.$$

Power? See Figure 10.1 and Theorem 10.6



"p-hacking"

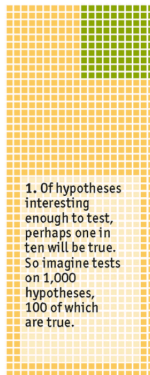


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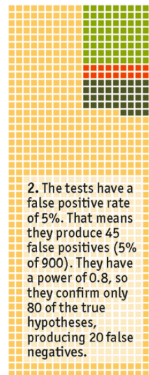
Unlikely results

How a small proportion of false positives can prove very misleading

False True False negatives False positives



1. Of hypotheses interesting enough to test, perhaps one in ten will be true. So imagine tests on 1,000 hypotheses, 100 of which are true.



2. The tests have a false positive rate of 5%. That means they produce 45 false positives (5% of 900). They have a power of 0.8, so they confirm only 80 of the true hypotheses, producing 20 false negatives.



3. Not knowing what is false and what is not, the researcher sees 125 hypotheses as true, 45 of which are not. The negative results are much more reliable—but unlikely to be published.

Source: *The Economist*

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→ Monash and Face Masks