

Mathematical Statistics II

STA2212H S LEC9101

Week 4

February 3 2021

Start recording!



Calling Bullshit @callin_bull · Jan 30

Guys. Time for some causal graph theory.

...



Hillary Clinton ✅ @HillaryClinton · Jan 28

Data proving @GretaThunberg right—"you are never too small to make a difference."

...



Geoffrey Supran @GeoffreySupran · Jan 26

"The @GretaThunberg Effect" is now an empirically demonstrated, peer-reviewed phenomenon:

"We find that those who are more familiar with Greta Thunberg have higher intentions of taking collective actions to reduce global warming."

The Greta Thunberg Effect: Familiarity with Greta Thunberg predicts intentions to engage in climate activism in the United States

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Abel Gustafson³  | Matthew H. Goldberg⁴  | Edward W. Maibach⁵  |
John E. Kotcher⁵  | Janet K. Swim⁶  | Seth A. Rosenthal⁴  | Anthony Leiserowitz⁴ 



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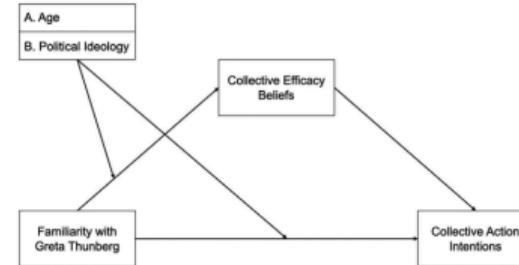


FIGURE 1 Conceptual model for hypotheses. Model tests the effect of familiarity with Greta Thunberg on intentions to take collective action through collective efficacy beliefs, as a simple mediation (Hypothesis 1), moderated by age (A; Hypotheses 2a and 2b), and moderated by political ideology (B; Hypotheses 3a and 3b), respectively



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- Choosing test statistics – optimality, likelihood, convenience

- Simple and composite hypotheses $\theta = \theta_0; \quad \theta \in \Theta_0$

- Generalized likelihood ratio tests $W(\theta_0) = 2\{\ell(\hat{\theta}) - \ell(\tilde{\theta}_0)\}$

- Wald tests $(\hat{\theta} - \theta_0)/\widehat{se}$

- significance tests $p\text{-value: } \Pr(T > t^{obs}; H_0)$

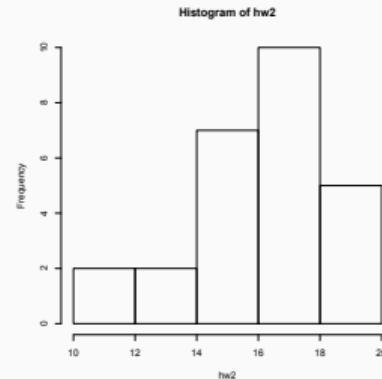
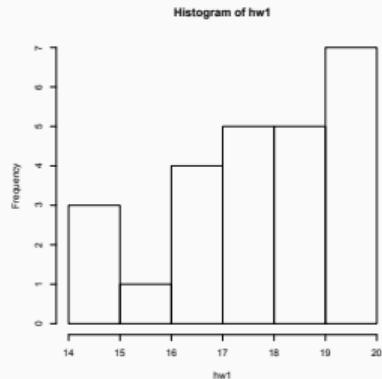
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1. Homework 2
2. confidence intervals
3. goodness-of-fit tests
4. multiple testing



- February 8 3.00 – 4.00 Paul McNicholas
- “Selected Problems in Classification” [Link](#)

Data Science and Applied Research Series



Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-34.103704	6.530014	-5.223	1.76e-07	***
zn	-0.079918	0.033731	-2.369	0.01782	*
indus	-0.059389	0.043722	-1.358	0.17436	
chas	0.785327	0.728930	1.077	0.28132	
nox	48.523782	7.396497	6.560	5.37e-11	***
rm	-0.425596	0.701104	-0.607	0.54383	
age	0.022172	0.012221	1.814	0.06963	.
dis	0.691400	0.218308	3.167	0.00154	**
rad	0.656465	0.152452	4.306	1.66e-05	***
tax	-0.006412	0.002689	-2.385	0.01709	*
ptratio	0.368716	0.122136	3.019	0.00254	**
black	-0.013524	0.006536	-2.069	0.03853	*
lstat	0.043862	0.048981	0.895	0.37052	
medv	0.167130	0.066940	2.497	0.01254	*

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

logistic regression

```
> summary(lm1)

Call:
lm(formula = log(cost) ~ date + log(t1) + log(t2) + log(cap) +
    pr + ne + ct + bw + log(cum.n) + pt, data = nuclear)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -14.24198   4.22880  -3.368  0.00291 ** 
date         0.20922   0.06526   3.206  0.00425 ** 
log(t1)       0.09187   0.24396   0.377  0.71025    
log(t2)       0.28553   0.27289   1.046  0.30731    
log(cap)      0.69373   0.13605   5.099 4.75e-05 *** 
pr            -0.09237  0.07730  -1.195  0.24542    
ne            0.25807   0.07693   3.355  0.00300 ** 
ct            0.12040   0.06632   1.815  0.08376 .  
bw            0.03303   0.10112   0.327  0.74715    
log(cum.n)   -0.08020  0.04596  -1.745  0.09562 .  
pt            -0.22429  0.12246  -1.832  0.08125 . 

---

```

log(cap)	0.69373	0.13605	5.099	4.75e-05	***
pr	-0.09237	0.07730	-1.195	0.24542	
ne	0.25807	0.07693	3.355	0.00300	**
ct	0.12040	0.06632	1.815	0.08376	.
bw	0.03303	0.10112	0.327	0.74715	
log(cum.n)	-0.08020	0.04596	-1.745	0.09562	.
pt	-0.22429	0.12246	-1.832	0.08125	.

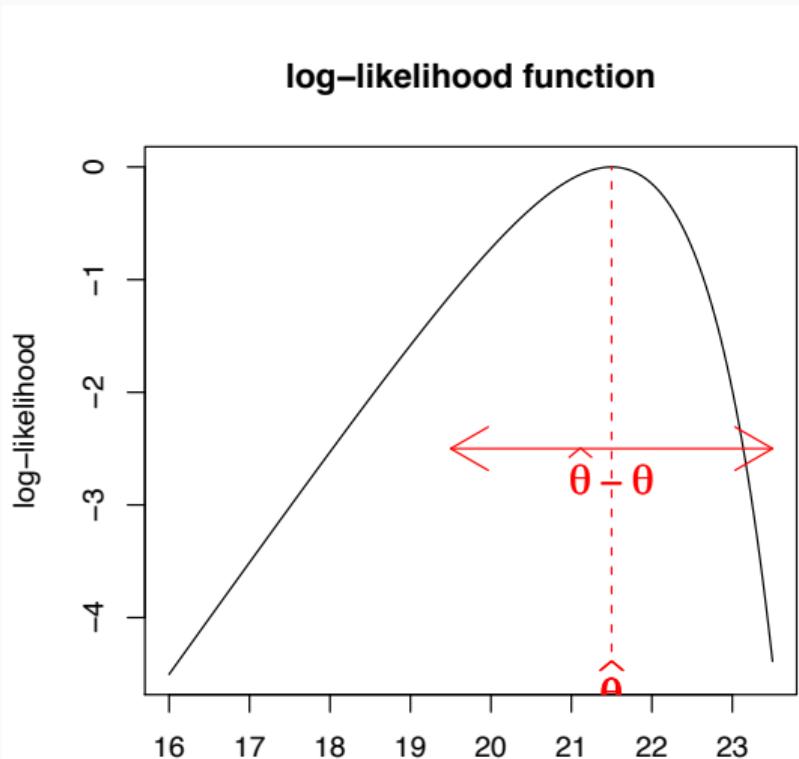
Residual standard error: 0.1645 on 21 degrees of freedom

Multiple R-squared: 0.8717, Adjusted R-squared: 0.8106

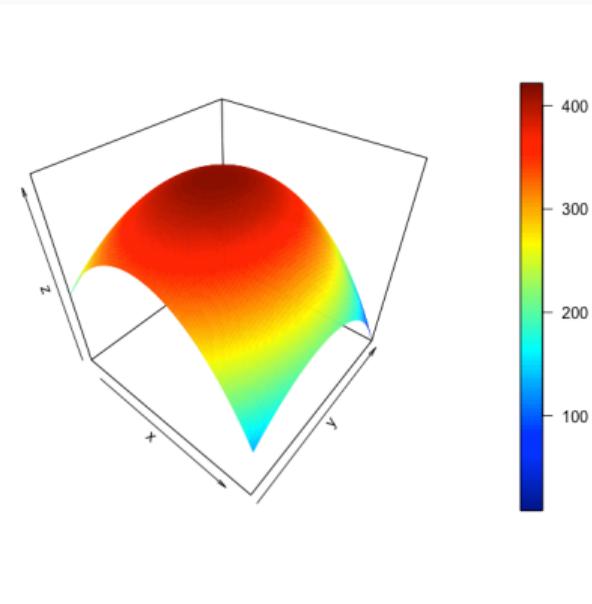
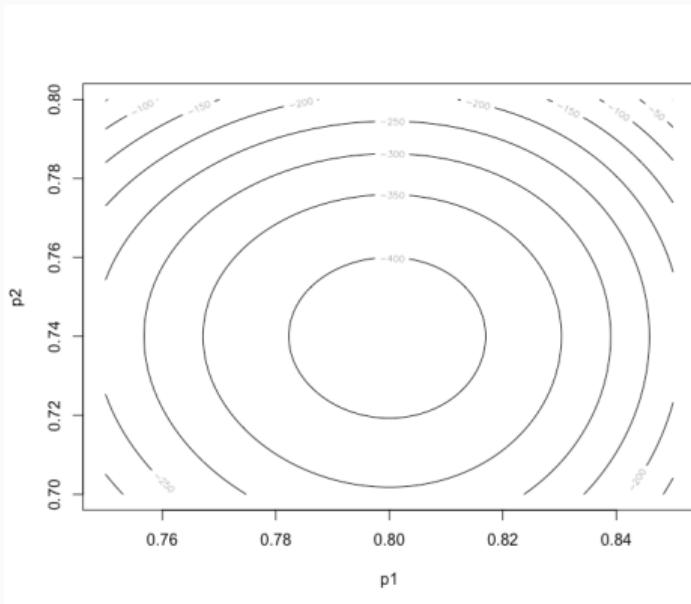
F-statistic: 14.27 on 10 and 21 DF, p-value: 3.081e-07

Example: Likelihood confidence intervals

... likelihood confidence intervals



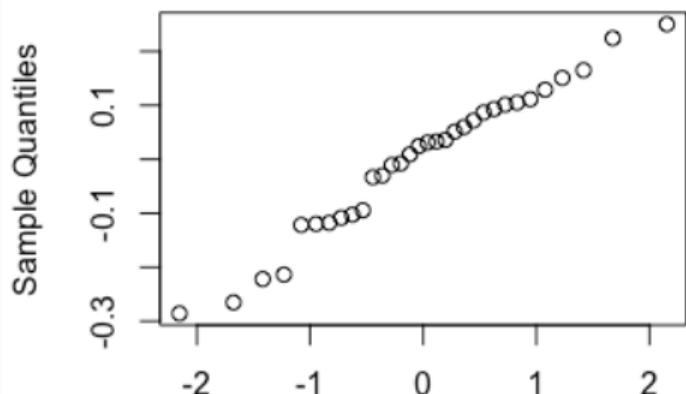
... likelihood confidence intervals



- interpretation of confidence interval or confidence bound random endpoint(s)
- size of test \leftrightarrow confidence level
- power of test \leftrightarrow width of confidence interval

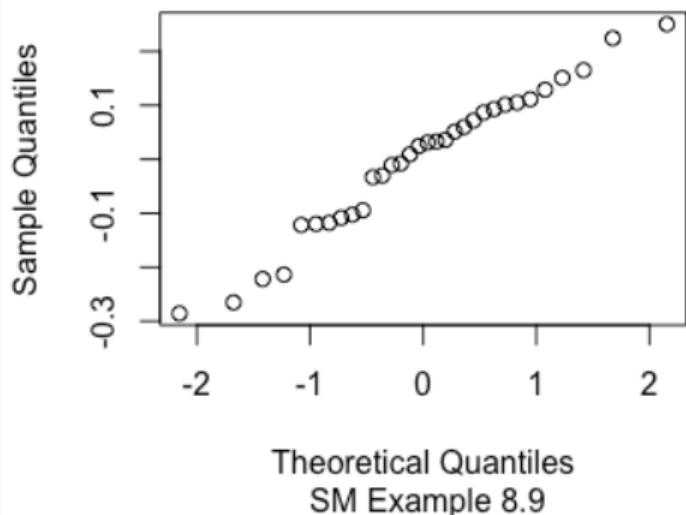
- data x_1, \dots, x_n independent, identically distributed
- test statistic $t = t(\mathbf{x})$, observed value t^{obs}
- $p^{obs} = \Pr(T \geq t^{obs}; H_0)$
- Example: sign test

residuals from linear regression



Theoretical Quantiles
SM Example 8.9

residuals from linear regression



Maize data SM Ex 7.24

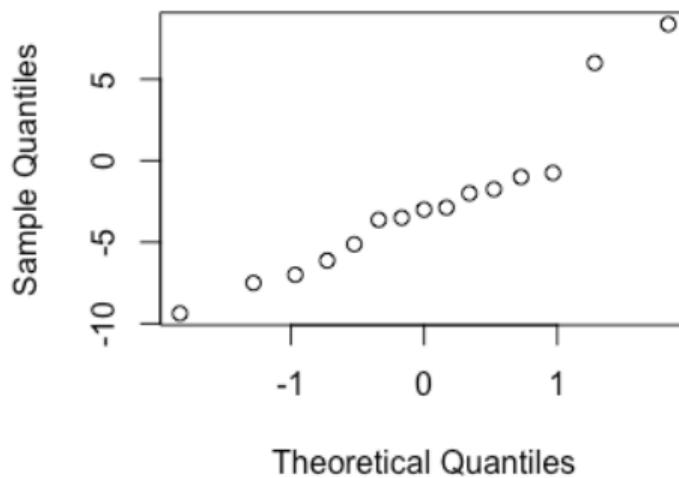
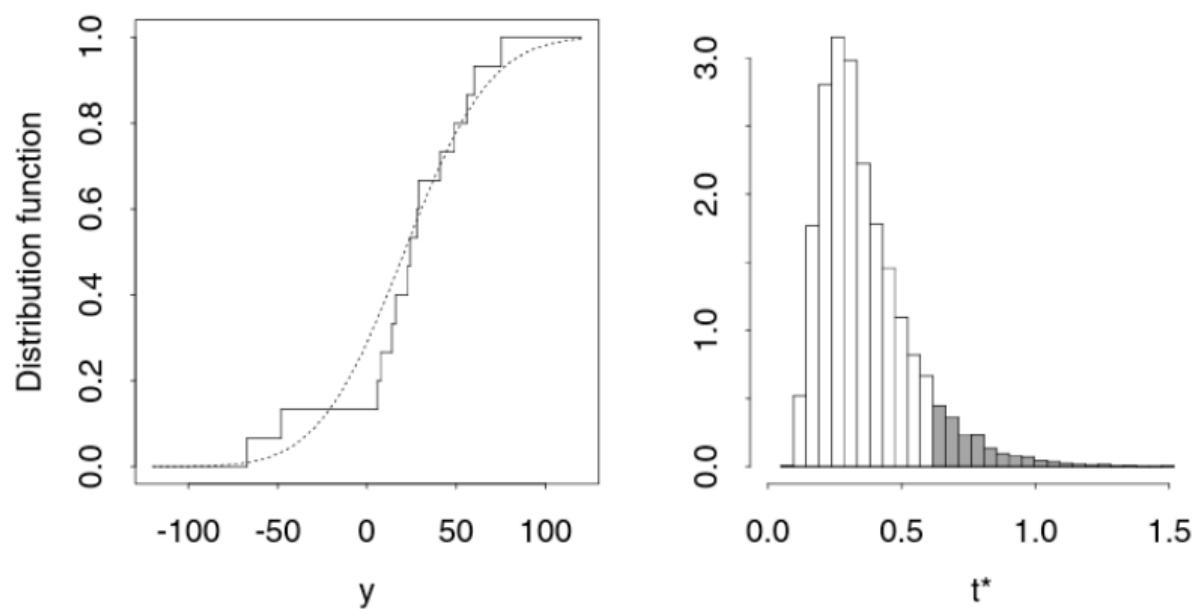


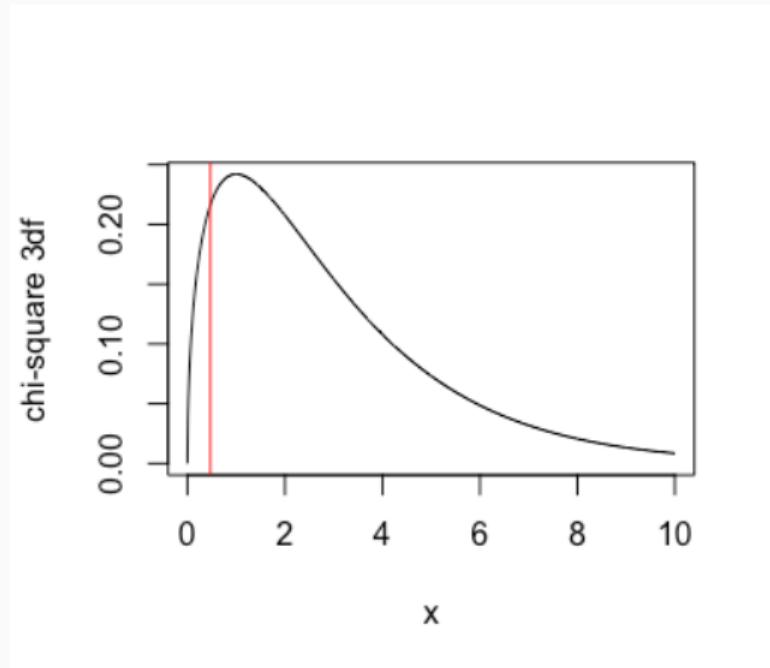
Figure 7.5 Analysis of maize data. Left: empirical distribution function for height differences, with fitted normal distribution (dots). Right: null density of Anderson–Darling statistic T for normal samples of size $n = 15$ with location and scale estimated. The shaded part of the histogram shows values of T^* in excess of the observed value t_{obs} .



SM Example 7.24 testing $N(\mu, \sigma^2)$ distribution

- X_1, \dots, X_n i.i.d. $F(\cdot)$; $H_0 : F = F_0$ cumulative d.f.
- $\hat{F}_n(t) = \frac{1}{n} \sum_{i=1}^n \mathbf{1}\{X_i \leq t\}$
- test statistic:
 1. $\sup_t |\hat{F}_n(t) - F_0(t)|$
 2. $\int \{\hat{F}_n(t) - F_0(t)\}^2 dt$
 3. $\int \frac{\{\hat{F}_n(t) - F_0(t)\}^2}{\hat{F}_n(t)\{1 - \hat{F}_n(t)\}} dt$
 4. χ^2 tests AoS 10.4
- SM Example 7.24 testing $N(\mu, \sigma^2)$ distribution
- SM Example 7.23; 6.14 testing $U(0, 1)$ distribution

AoS Example 10.18



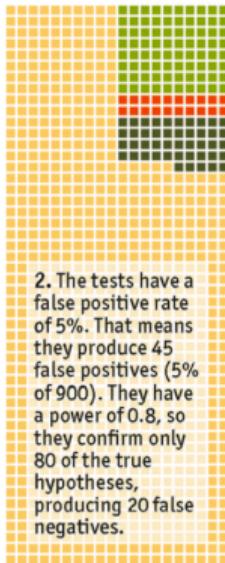
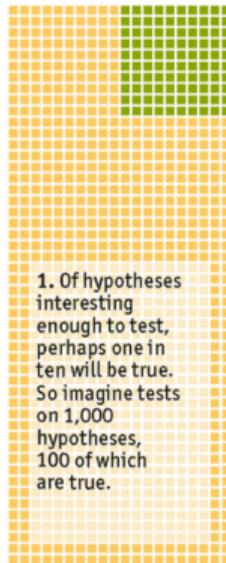
Pause

Multiple testing

Unlikely results

How a small proportion of false positives can prove very misleading

■ False ■ True ■ False negatives ■ False positives



Source: *The Economist*

False Discovery Rates

Benjamini-Hochberg

Benjamini-Hochberg

